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ABSTRACT

The New Teachers Mathematics Staff Development Program for grades 9 through 12 is a combination of content and methodology. Teachers focus their attention on content, lesson planning, questioning, homework, and testing. This volume contains materials for eight workshop sessions. Sessions included are: (1) An Introduction to the New York City School System; (2) Managing the Mathematics Classroom; (3) The Curriculum; (4) Planning and Executing an Effective Mathematics Lesson; (5) Questioning in the Mathematics Classroom; (6) Developing Reasoning Skills, Critical Thinking, and Problem Solving Ability in the Mathematics Classroom; (7) Homework Planning and Review To Maximize Student Learning; and (8) The Teacher's Role in Student Assessment. The sections contain background materials, outlines, teaching tips, activities, and evaluation materials; materials provided in each section vary. (RH)

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Staff Development for New Teachers

MATHEMATICS

High School

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FOREWORD

By making the decision to join the ranks of Mathematics educators of New York City, you have become part of a highly respected family of colleagues with a long tradition of excellence and professional commitment. I welcome you into this circle and remind you of the readiness of the Division of Curriculum and Instruction to make your entry into your classroom as smooth as possible.

There are many people who stand ready to help you, including your fellow teachers, your department supervisor, your principal and the staff of your Superintendent's office. As an introductory orientation to the responsibilities and joys of teaching Mathematics, this Staff Development Program for New Teachers will provide you with a detailed overview of the concerns of Mathematics teachers -- one that I believe you will be referring to time and again, even when you are no longer a new teacher.

In a sense, we are always "new" teachers. There is something about this profession that renews us every semester: the challenge of new youngsters, the challenge of new curricula, and the challenge of new personal goals for achievement. It is rewarding for me to be participating in your introduction to this exciting world, and I extend my very warmest wishes for every success.

Sincerely,

Charlotte Frank

Charlotte Frank
Executive Director,
Division of Curriculum and Instruction

ACKNOWLEDGMENTS

The New Teacher Mathematics Staff Development in Grades 9 through 12 Mathematics was developed as a project of the Mathematics Unit of the Division of Curriculum and Instruction, Charlotte Frank, Executive Director.

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INTRODUCTION

The New Teachers Mathematics Staff Development for Grades 9 through 12 is a combination of content and methodology. It is important to have a knowledge of what to teach as well as how to teach. Teachers will focus their attention on content, lesson planning, questioning, homework and testing. Teachers must become proficient in these essential areas in order to deliver effective instruction.

As participants master these significant aspects of teaching, a central thread runs through these areas. Student verbalization allows students to become active participants in the process of teaching and learning. Research has shown that expressing one's thinking through verbalization brings thinking to the conscious level. Students who take a verbally active part in each lesson through discussing, questioning, explaining and justifying their solutions have allowed their thoughts to be formulated and polished through the additional sense of hearing. Students who are asked to verbalize what strategies they have used to solve a set of problems are more successful on a subsequent set of related problems than students who have not been asked to put their strategies into words.

It is essential that teachers totally involve students in all aspects of the learning process. The teacher must have students respond during class in full sentence answers so that they are reinforcing not only what is done to solve a problem but also when and how the method is to be applied. The teacher must have students come to the board, write on the board and verbalize the strategies for solutions to the homework, class problems and teacher-made tests. Mathematics instruction becomes most effective when students actively discuss what they are learning.

Through verbalization, students see mathematics as a close knit system of understandable ideas, principles and processes. Students are more likely to retain mathematical learning over time if meaningful discussion activities comprise more than half of their mathematics instruction. Teachers must aim for a regular and consistent involvement of students through verbalization in order to assure meaningful instruction in mathematics. The crucial relationship between student verbalization and understanding is essential to insure student retention.

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NEW TEACHER STAFF DEVELOPMENT

Grades 9-12

WORKSHOP OUTLINE

MATHEMATICS

Session Title: An Introduction to the New York City School System.

The teacher will be able to:

1. Demonstrate an understanding of school policies, procedures, and regulations.
2. Identify records, reports, and forms used in the secondary school.
3. Organize a classroom for instruction.
4. Identify materials and supplies needed for the first week of school.
5. Explain the roles and responsibilities of the secondary school teacher.
6. Demonstrate an understanding of high school/local community school district organization.

Session Title: Classroom Management.

The teacher will be able to:

1. Identify the basic routines necessary to manage the class.
2. Develop classroom rules and routines and teach these to students.
3. Establish techniques for dealing with discipline problems.
4. Identify appropriate strategies for handling students with special needs.
5. Participate in simulations reflecting classroom management problems.
6. Develop strategies for improving student attendance.
7. Describe strategies that may be used to reduce mathematics anxiety.

Session Title: Planning and Executing an Effective Mathematics Lesson.

The teacher will be able to:

1. Utilize grade and course specific New York City Scopes and Sequences and Curriculum Bulletins.
2. Identify the significant aspects of a mathematics lesson.
3. Identify the resources that may be used in planning a mathematics lesson.
4. Explain strategies that may be used in executing a lesson to increase student retention of mathematical concepts and facts.
5. Explain strategies that may be used to maximize the instructional time in the classroom.
6. Describe techniques of motivation.
7. Evaluate sample lesson plans.

Session Title: Questioning in the Mathematics Classroom.

1. Explain the role of questioning in the mathematics classroom.
2. Identify types of questions to avoid asking in a mathematics classroom.
3. Explain ways of formulating questions to enhance student learning.
4. Identify common mistakes in questioning used in the mathematics classroom.
5. Develop criteria for judging questions asked in the classroom.

Session Title: Developing Reasoning Skills. Critical Thinking, and Problem Solving Ability in the Mathematics Classroom.

The teacher will be able to:

1. Describe the importance of problem solving in mathematics.
2. Describe methods that teachers can employ in the classroom to assist students in their development of problem solving skills.
3. Identify problem-solving strategies that can be taught to students to help them become better problem solvers.
4. Assess their classroom atmosphere to determine if it fosters problem-solving and critical thinking for students.
5. State questions that may be used to encourage mathematical thinking.
6. Develop quidelines for establishing a positive classroom environment for problem solving.

Session Title: Homework Planning and Review to Maximize Student Learning

The teacher will be able to:

1. Explain how to plan homework using a cyclic approach.
2. State the benefits of planning cyclic homework assignments.
3. Develop strategies for students on how to do mathematics homework to maximize student learning.
4. Describe methods of reviewing students' homework.
5. Identify characteristics of a poor homework assignment.

Session Title: The Teacher's Role in Student Assessment.

The teacher will be able to:

1. Describe the purposes of testing within the mathematics classroom.
2. List the steps to be taken in the planning of tests.
3. Describe the procedures/suggestions for giving a mathematics test, the steps to be taken in marking a mathematics test, and the procedures/suggestions for going over a mathematics test with students.
4. Describe strategies to help students taking mathematics tests.
5. Describe ways of evaluating a test and interpreting test results.
6. Explain how to evaluate student performance in mathematics.

Session Title: Simulation and Summation.

The teacher will be able to:

1. Write, critique and role play lesson components.
2. Explore additional areas of interest or topics in mathematics.
3. Discuss any topics or activities utilized during the previous six days of training.
4. Develop some sample flowcharts, dialogue and guidelines for the first week of classes.
5. Suggest additional topics for the follow-up days (Days 8, 9, 10) for New Teacher Training.
6. Summarize recent articles from the Mathematics Teacher and the Journal of Research in Mathematics.

THE TEACHER

A Model for Adolescents

Teenagers want their teachers to be adults. Although they test their teacher's relationships with them, they expect their teachers to carry themselves as representatives of something larger than the immediate world of pop culture or the familiar world of their own friendships. In a world in which inspiration, guidance and positive values are often overshadowed by the bitter realities of existence, the teacher can be one of the most important persons in a youngster's life.

It is a misinterpretation of the teacher's role to see it as an opportunity to participate in adolescent activity in order to be accepted by adolescents. Respect for a teacher's adult point of view, developed through the teacher's own experience and education, is far more important than "chummy" recognition. Courts have described the teacher's position in certain situations as being in loco parentis -- "in place of the parent". The last thing a youngster needs from his teacher is one more person to whom to relate on the same levels on which he relates to his friends. What the youngster does need is a grown-up who will represent the highest values of society and education -- something to which to aspire, a good person to emulate.

The opportunities to express the special role of the teacher in the life of a youngster are many. Not all of them can be foreseen. It is often the spontaneous reaction of a teacher to a situation that leaves an indellible mark, positive or negative, in the memory of a youngster. Some opportunities, on the other hand, can be foreseen and methodically pursued. A teacher's appearance and dress can be designed to reflect his or her adult, professional position in society. A teacher's speech and choice of vocabulary can be made free of vulgarity. A teacher's personal department with students can be kept free of familiarity or intimacy, free of easily misunderstood verbal expressions and physical contact.

A teacher's reactions to the minute-by-minute unfolding of events during the school day can be monitored with respect to avoiding inappropriate anger, excessive fault-finding and criticism of people, devaluation of education or schooling, and disrespect for authority. When "something happens", youngsters will be looking at their teachers to see how to react. They can learn, from these examples of adult behavior, to emulate whatever model we present to them for emulation. In the larger world outside school, especially in the entertainment media, examples of hedonistic, violent,

vengeful, sensual and irresponsible behavior abound and are often presented in association with characters who are made to appear worthy of emulation. The teacher is one of society's few champions of another view of life: the reality of responsibility to one's fellow humans and to one's own potential, the abhorrence of violence, the recognition of higher pursuits than the mere satisfaction of appetites, the distinction between true happiness and simple pleasure. Overriding all of this is the teacher's commitment to the value of education as the vehicle for developing these values.

The Colleagueal Relationship

A school is an organic, dynamic set of relationships between the people who make it up: the teachers, the students, the parents, the supervisors and administrators. More than the building and its physical features and equipment, the school is this connective tissue between people. If -- heaven forbid! -- the building weren't there one Monday morning, the school, as embodied in these relationships, could still be alive and well.

Central to this fabric of interrelationships is the collegueal relationship between teachers. We are not a buildingful of isolated practicioners, each "doing our own thing". Our responsibilites extend to include the maintenance of supportive, respectful and sharing relationships with our fellow-teachers. For us to do our best job, we recognize our dependence on each other's special knowledge and experience. In this respect, teaching is different from other fields of activity in which competition and virtuosity are held in highest esteem. Our relationship to each other is closer to that of members of professions in which mutual respect, support and dependence is taken for granted and is regarded, in turn, with the highest respect by society in general.

Every teacher in the system is the teacher's colleague, since we are all working, through our own specialties, towards the same general principles. From this point of view, distinctions between teachers of different subject matter, teachers in different departments or in different schools, aren't nearly as important as the things that make us colleagues: our shared belief in the value of human potential and the central role that education plays in helping people to live up to it.

Teachers can help each other in many ways. More experienced teachers can help less experienced teachers to prepare lessons. Teachers can observe each other's teaching. Teachers can share copies of materials that they have prepared with each other. We can pay special attention to adhering to courses of study and calendars of lessons, since deviation from them leads to differences in the way students are prepared

and invites comparision between the work of different teachers of the same subject. We can make it a point to complete paper work accurately and one time, since failure to do so invariably means the creation of problems for colleagues who are waiting for it.

Everyday we can add to or subtract from this ideal relationship between colleagues. When we "put down" another teacher in speaking to students, when we elevate ourselves above other teachers, when we fail to cooperate with fellow teachers, we tear down this relationship. When we share with fellow teachers, when we come to their aid, when we speak of them in a way that elevates all teachers, we are building that relationship. Teaching, with a capital "T", benefits when students, parants and teachers observe this positive relationship in operation.

Professional Organizations

One of the best ways to become even more widely involved with colleagues is to join and be active in professional organizations. Through the publications of these organizaions, you can keep abreast of developments in the field of mathematics education as well as on-going discussion of issues that effect mathematics education. Through meetings and conferences that the organizations provide, teachers can interact with creative and enthusiastic colleagues in workshop experiences. As a teacher gains increased experience and special knowledge, he or she may become a speaker or workshop leader through an organization's invitation. The major professional organizations for mathematics educators are listed below.

ORGANIZATION	ADDRESS	CONTACT PERSON
National Council of Teachers of Mathematics (NCTM)	1906 Association Drive Reston, VA 22091	
Association of Teachers of Mathematics of New York City (ATMNYC)	P.O. Box 355 Thiells, NY 10984	Ms. A. Cacciola
Association of Teachers of Mathematics of New York State (AMTNYS)	R.D. Box 49 Coeymans Hollow, NY 12046	
Standing Committee of Mathematics	Mr. Noel Gurin Springfield Gardens H.S. 143-10 Springfield Blvd Springfield Gardens, NY 11413	Mr. Noel Gurin A.P. Math

Student Behavior

The teacher's greatest ally in getting students to exhibit their best behavior is an interesting, engaging activity for them to be involved with. Well-prepared lessons that stress student participation in a variety of ways (working with manipulatives at seats, working at the board, role-playing, making presentations, etc.) go a long way towards eliminating the boredom and inactivity that so often lead to disruptive student behavior. We distinguish between "busy work", that is, repetitive, time consuming work for no apparent purpose, and engaging work -- work that is perceived as interesting and important by students. We can do a lot to broadcast our own attitudes about particular bits of work, and students are quick to pick them up. When we repeatedly introduce exercises with the invitation "Now let's do another one" we are charactersing the work as an endless procession of inherently uninteresting exercises. When we preface new material with apologetic expressions such as "Now here's a hard one" or "There's a fancy word for this ..." we are prompting students to regard what is coming as something negative. When every lesson begins in exactly the same way, with a routine, unengaging "Do Now" on the board, we are setting up low expectations for the lesson to follow.

Of course, the most engaging of teachers is going to have instances of behavior problems. Often, these are expressions of problems that students are harboring and about which we are completely ignorant. Youngsters these days are walking around with burdens that often overshadow anything that school may have to offer by way of diversion. Many students live in one-parent homes. Others are living in homes that are about to become one-parent homes. Many students have financial responsibilities after school and/or before school that may render them tired and inattentive. Even youngsters who do not have problems of this magnitude are burdened by the normal but considerable problems of adolescence. Many of them are insecure, over-sensitive, and easily embarrassed. It is safe to assume that, in some way, they are all at least a little worried that they are not good enough and are trying desperately to prevent the world from finding this out. Little wonder that, as they make their way through a day in school, interacting with seven or more teachers and hundreds of fellow students, so many of them exhibit unacceptable behavior.

Each teacher can at least see to it that he or she provides no stimulus for triggering bad behavior. We avoid, for instance, remarking unfavorably (even in jest) about a youngster's appearance, because this is one of the surest ways of eliciting a bad response. We avoid public verbal exchanges with youngsters, since these often evoke the face-saving instinct that leads to escalated "one word led to another" situations. We avoid making emotional outbursts in anger, especially raising our voices, because these inspire like behavior in students.

On the other hand, we can do much to elicit the best from youngsters by communicating that we respect them and are sensitive to problems. Sincerely complimenting students for good work helps to create a positive reward that most students will want to work for. Inquiring about a student's health after an illness and providing for a reasonable way for making up missing work or exams can go a long way towards communicating the teacher's concerned decency and eliciting a like response. Giving students the benefit of the doubt often calls up their best cooperation.

Wherever possible, behavior incidents should be handled by the teacher, if not at the moment, then after class. Habitually resorting to such methods as expelling a student from class (which is illegal), sending a student to the department chairman (which is not generally appreciated) or to the dean's office communicates very clearly to students that you are incapable of handling situations yourself. These methods undermine the development of the teacher's own ability to deal with difficult situations.

With increased experience, the teacher learns to distinguish between run-of-the-mill incidents which he or she can handle effectively without help and special incidents which are indicative of very serious problems. In such cases we welcome the help that colleagues can give. Drug-related problems, problems that involve weapons, fighting, theft or other crime, problems that may stem from child-abuse situations, problems that may relate to undetected physical or psychological disorders -- all of these are situations in which the full range of school specialists should be enlisted. Consultation begins with the department head and may then involve the guidance counsellor, the SPARK coordinator, or outside agencies. Often what is manifested as poor behavior is really an undetected learning disability that is best brought to the attention of Special Education specialists.

The First Few Days

The first few days of a new school year, semester or cycle will seem like a movie run at three times the normal speed. You will wonder where the hours went and how anyone is expected to teach anything to anyone under conditions that may include any or all of the following: students dropped and added to your classes at the worst possible moments, textbooks running short when you least expect it, chalk, erasers, book receipts, delaney cards running out when you most need them, faculty and department conferences scheduled when you have the least amount of time for them, students approaching you with the discovery that they are in the wrong class, etc., etc. It is a trying time for teachers, but also an exciting one as you anticipate teaching new groups of youngsters and, perhaps, new subject matter. The object is to get through these first few days with as little unexpected difficulty as possible, and knowing what to anticipate can help you to avoid much of this. Here are some suggestions to follow before the first few days of teaching:

* Ask your supervisor for copies of the grading policy, the attendance and cutting policies.

* Familiarize yourself with the school schedule, noting the beginnings and endings of periods and the start and end of teacher time for the school day.

* Learn the procedure for calling in sick. Check with your supervisor as to the procedure for leaving materials he/she can use for your replacement when you are absent.

* Locate your mailbox, both in the general office and in the department office, if one has been assigned to you.

* Obtain the following supplies: chalk, eraser, book receipts, delaney cards, delaney book.

* Obtain copies of the courses of study for the classes that you will be teaching, including published curriculum guides if they exist for your courses.

* Find out the names of other teachers who will be teaching the same courses that you will be teaching.

* Find out if a grade leader has been designated for your courses and what your duties are with regard to the grade leader.

* Obtain keys to the rooms in which you will be teaching, the department office, if this is the policy in your school, and the teacher's bathroom.

* Inquire of your supervisor what the preferred procedures are for taking attendance, reporting cutting, filling out delaney cards, distributing textbooks, collecting homework, etc.

* Learn from your supervisor acceptable formats for lesson plans. Set up a lesson plan notebook and plan the first several lessons, leaving time for administrative matters.

* Become aware of specific goals related to your school and the mathematics department regarding the mathematics program (i.e., improving regents examination results, improving student attendance, improving reading and writing skills) and relate them to your class instruction.

* Familiarize yourself with the "chain of referral" in your school, should problems arise.

* Familiarize yourself with the physical layout of your school, noting the locations of main offices.

* Remember, above all else, that students don't need to know how disoriented you may be. They are expecting you to be a source of stability and reassurance. Especially if they are also new to the school, they are even more in need of these than you are.

Before things are too far under way, establish classroom routines which will facilitate the optimum use of instructional time. Inform students right from the start what will be expected of them in terms of work and classroom behavior. Remind them about attendance and lateness regulations. Explain to them what polices will be followed in formulating their grades, and make sure that whatever policies you adopt are consistent with department, school and High School Division policies. Whatever policies are being used, the data upon which grades will be based must be documented in writing, either in the form of notations on delaney cards, in a marking book or on forms of your own design. If class participation is an element to be considered in formulating grades, then class participation must somehow be recorded. It is embarrassing for the teacher and supervisors when a grade is questioned and no documentation can be provided to back it up.

Establish routines for collecting and returning homework papers and exam papers, and get into the habit of using students to do classroom chores like erasing the boards and distributing and collecting papers. Instruct students regarding the important opening few minutes of each lesson, including the manner of presenting homework at the chalkboard and working on an opening activity.

One of your primary goals in these routines is to make it possible for you to use as much time as possible for instruction. In a forty-minute period, each minute represents 2.5% of the available time. If excessive time is spent on reviewing homework, passing papers or calling the roll, little time will be left for teaching and learning. Develop quiet and efficient ways of checking attendance without using instruction time for this, waiting until students are busy at work at their seats. Learn how to postpone necessary student conferences until the end of the class instead of interrupting instruction to discuss a student's individual problems with attendance, lateness, etc. "Please see me after class" can be an important time-saver for you and your students. This also will avoid embarassing public confrontations.

Class Participation

Students have different ways of learning best, and different ways of expressing what they have learned. Some have no trouble writing what they know under pressure of time, and, therefore, do very well on written exams. Others have a great deal of difficulty doing well on written activities including exams, but are very verbal and articulate when they have the opportunity to speak about what they know. Classroom activities should be varied enough to allow all students to excel in expressing what they know in a variety of ways. If all students were held to a grading policy that allowed only for written performance, many able students would be doomed to failure. Thus, class participation is more than merely a way of enlivening and "socializing" the classroom, although it certainly does do these things. It is an important tool in the teacher's job of finding out what students know.

The way in which class participation develops in a class is an outgrowth of the teacher's questioning skills, especially with regard to his or her reaction to student responses to questions. This aspect of teaching requires the greatest skill, and one should, therefore, begin to work towards mastery in this area right from the start. How class discussion develops can either encourage or kill critical thinking, give confidence or rejection to students, facilitate or impede the development or problem solving skills, provide for or rule out the development of reasoning skills. Without going into a detailed account of questioning skills here, suffice it to recall that the most important word in the teacher's vocabulary is "why" and the greatest challenge is getting students to think more and say more.

The greatest sensitivity with regard to class participation is required when dealing with students for whom English is not the first language. Already self-conscious about expressing themselves in English, expressing themselves regarding the abstractions of mathematics poses a double threat. Yet, to leave them unchallenged to try to do so would be to do them a disservice, while forcing the issue to the point of embarrassment is also doing them no favor. The goal should be, as with all students, to take them from where they are to a level of expression that is more than they would have achieved without your intervention.

Conditions in your classroom may be very special with regard to this situation. You may have large numbers of students all with similar language backgrounds, in which case the preparation of mathematics glossaries, with student input, may be helpful in providing an on-going reference for recurring vocabulary words. On the other hand, your classroom may contain a great variety of language backgrounds, in which case you may want to confer with colleagues who can help with specific needs.

An apparent paradox of mathematics education is that students who may exhibit nearly total inability in English can be extremely proficient, even superior, in mathematics. Of course, this is no paradox. Simply imagine yourself transported to a country in which you are illiterate or barely literate and consider what effect this would have on your mathematical ability. The greatest effect would be on your ability to express your mathematical ability, not on your ability itself.

Fortunately, mathematics is a language unto itself, and the mathematics teacher can capitalize on this by providing students with the technical vocabulary they need to express the understandings which, in many cases, they already have. Mathematics, because of its universality, is often the one class in which students having difficulty with English can excel. It is everyone's second language.

The High School Division and the Community School Boards

It is important for a teacher to understand the position that his or her school occupies in the larger structure of the New York City school system. All too often teachers think in terms of "The Board" or "The System" without understanding who it is they really should be referring to.

Your High School

The New York City high schools are administered by the High School Division of the Board of Education. This is a centralized organization, with this one division setting policy for all the high schools in the city. Thus, all the various and many school policies regarding attendance, grading, curriculum, etc. must be designed so that they are consistent with High School Division policy.

To facilitate administration (there are some 111 high schools in New York City) the high schools are further subdivided geographically into five superintendencies, each with its own Superintendent of High Schools. These follow the borders of the five boroughs, except in the case of BASIS (Brooklyn and Staten Island Schools), which consists of the high schools of Staten Island and some from Brooklyn. The superintendent is the principal's direct supervisior. Become familiar with broad goals of the high school division related to mathematics instruction. Take advantage of staff development programs that are offered centrally and through your district superintendent. Your supervisor will be able to assist you with this.

Your Intermediate School:

If you are teaching in a junior high school or intermediate school, your school is part of a Community School District, of which there are 32 in the city. Each district has its own administrative staff, headed by a superintendent and staffed by a variety of specialists who perform many of the same functions for the elementary and intermediate schools in the district that the High School Division performs for the high schools of the city. This is a more de-centralized organization, with many of the policies determined at the community level. Ultimately, the Community School Boards are under the umbrella structure of the New York City Board of Education, and their schools participate in the full range of central activities, but in many areas they exercise a greater degree of autonomy than individual high schools do.

It is important to become familiar with the community school district's resources that are available for mathematics instruction. Make an appointment to meet the District Mathematics Coordinator. He or she is a resource person who can assist you be providing additional resources and supplies.

It will enhance your professional growth to participate at conferences with other mathematics teachers. This will allow you to stay current in the field of mathematics as well as in the area of effective instruction. Through your District Mathematics Coordinator you will be able to learn about new curriculum manuals to meet New York City and New York State guidelines as well as district or citywide staff development programs.

THE TEACHER'S PLACE IN THE MATH DEPARTMENT ORGANIZATION

1. **Responsibilities to students**
   - A. Set a good example to pupils in your dress and appearance, your manners and self-control, your speech, your social attitudes and your personal evidences of character.
   - B. Display concern for the progress and welfare of your pupils.

2. **Responsibilities to colleagues**
   - A. Professional courtesy, consideration, cooperation.
     1. Arriving punctually for classes, special assignments, section and proctoring assignments.
     2. Leaving classrooms in suitable condition for colleagues who follow (blackboards, floors, etc.).
     3. Returning audio-visual equipment and other devices and materials promptly and in food order.
     4. Assuming fair share of non-teaching department responsibilities (book collection and distribution, grade chairmanships, preparation of uniform examinations, serving on curriculum committees, marking committees, etc.).

3. **Responsibilities to supervisor**
   - A. Carrying out policies and practices as outlines in administrative directory, circulars, conferences, etc.
   - B. Following suggestions made by supervisor.
   - C. Keeping supervisor informed of what you are doing.

4. **Responsibilities to self**
   - A. Interest in improvement.
   - B. Professional attitude towards job.
   - *** C. Membership in professional organizations, taking advanced courses, attending institutes, conferences, seminars, etc.

*** Keep in mind the benefits of belonging to professional organizations:

1. the involvement of teachers in problems and issues of common interest and concern
2. helps the teacher keep abreast of current developments in the field and publish contributions and critical reactions.

<u>Professional Associations for Mathematics Education</u>

National Council for Teachers of Mathematics (NCTM)
1906 Association Drive
Reston, VA 22091

Association of Teachers of Mathematics of New York City (ATMNYC)
P.O. Box 355
Thiells, NY 10984

Association of Teachers of Mathematics of New York State (AMTNYS)
R.D. Box 49
Coeymans Hollow, NY 12046

Situations for Discussion by New Teachers: Introductory Session

1. The teacher who occupies your room the period before you habitually keeps the class in beyond the end of the period, causing your class to start late each day. What do you do?

2. Between classes, a student passes you in the hall and verbally threatens you with physical harm. What do you do?

Consider the following interaction between a teacher and some students.

Teacher: Before we begin today's lesson, I want to remind you of the rule for participaton. Raise your hand before speaking.

Student A: (Without raising his hand.) You mean, if I have a question I need to raise my hand?

Teacher: That's correct.

Student B: (Without raising her hand.) What are we going to do today?

Teacher: I'll outline the lesson in a minute.

Student C: (Without raising his hand.) I thought you said we had to raise our hands?

Teacher: You're supposed to.

What are the consequences of the above?

Building Cooperation in Classroom Management

Items to Avoid:

1. A preoccupation with one's authority may provoke rather than stiffle defiance and resistance to discipline. The teacher should not be concerned with her own prestige.

2. Refrain from nagging and scolding, since it may fortify the child's mistaken concept of how to get attention.

3. Do not ask a child to promise anything. Most children will promise to change in order to get out of an uncomfortable situation. It is a sheer waste of time.

4. Do not give rewards for good behavior. The child may then work only in order to get his reward and stop as soon as he has achieved his goal. What's more, this will only strengthen his belief that he must be paid every time he acts civil or makes a contribution.

5. Refrain from finding fault with the child. It may hurt his self-esteem and may discourage him.

6. Avoid double standards - one for yourself and another for the students. In a democratic atmosphere everybody must have equal rights. This includes the chewing of gum, swearing, tardiness, unnecessary visiting, and talking with members of the faculty in class when the children are working, sitting when the class pledges allegiance, checking papers or doing any kind of work that prevents the teacher from looking at the child when he is talking to her.

7. Do not use threats as a method to discipline the child. Although some children may become intimidated and conform for the moment, it has no lasting value since it does not change their basic attitudes.

8. Do not be vindictive; it only stirs up resentment and unfriendly feelings.

Items to Consider:

1. Because problem behavior is usually closely related to the child's faulty evaluation of his social position and how he must behave in order to have a place in the class group, the teacher's first concern must be to understand the purpose of his behavior. Only then will the teacher be in a position to plan more effectively for this child.

2. Give clear-cut directions for the expected action of the child. Wait until you have the attention of all class members before you proceed in giving directions.

3. Be more concerned with the future behavior of the child rather than with past behavior. Refrain from reminding the child what he/she used to be or do.

4. As soon as a child misbehaves and tends to threaten the general atmosphere in the class, give him the choice either to remain in his seat without disturbing the others or to leave the classroom if possible or go to the back of the room.

5. Build on the positive and minimize the negative. There is much good in every child, but if you look only for academic achievement, you may never find it.

6. Try to establish a relationship with the child built on trust and mutual respect.

7. Discuss the child's problem at a time when neither of you is emotionally charged, preferably in the regular class discussions.

8. Use natural consequences instead of traditional punishment. The consequence must bear a direct relationship to the behavior and must be understood by the child.

Items to Consider:
(continued)

9. Be consistent in your decisions. Do not change a decision arbitrarily just because it suits your purpose at the moment. Inconsistency confuses the child about what is expected of him at a certain procedure.

10. See Behavior in its proper perspective. In this way, you will avoid making a serious issue out of trivial incidents.

11. Establish cooperative planning for future goals and the solution of problems.

12. Let children assume greater responsibility for their own behavior and learning. They cannot learn this unless you plan for such learning. Teachers who are afraid to leave the room because of what might happen prevent the children from taking responsibility. Responsibility is taught by giving responsibility. Be prepared for children to act up at first. Such training takes time.

13. Use the class as a group to express disapproval when a child behaves in an antisocial manner.

14. Treat the child as your social equal.

15. Combine kindness with firmness. The child must always sense that you are his friend, but that you would not accept certain kinds of behavior.

16 At all times distinguish between the deed and the doer. This permits respect for the child, even when he does something wrong.

17. Guide the individual to assume independence and his own self-direction.

18. Set the limits from the beginning, but work toward mutual understanding, a sense of responsibility, and consideration for others.

19. Admit your mistakes - the children will respect your honesty. Nothing is as pathetic as a defeated authoritarian who does not want to admit her defeat.

20. Mean what you say, but keep your demands simple, and see that they are carried out.

21. Children look to you for help and guidance. Give them this security, but make cooperation and eventual self-control the goal.

22. Keep in mind your long-term goal: an independent, responsible adult.

23. Children need direction and guidance until they can learn to direct themselves.

24. Close an incident quickly, and revive good spirits. Let children know that mistakes are corrected and then forgotten.

25. Commend a child when his behavior in a situation shows improvement.

26. Work cooperatively with the children to develop a procedure for dealing with infractions of the rules.

27. "Do unto others as you would have them do unto you."

As teachers we need to exercise kindness but also firmness so that the children will know what to expect from us; thus mutual respect between students and teachers will be the result of the democratic educational process. In this way the child will be able to project the inner order that exists in him out into his environment and will be able to make order in his own way, find his own place, and develop self-discipline.

When the students understand what is expected of them, when they have been accepted and respected as equal partners, when the teachers no longer feel threatened, then and only then, will they be ready to move into the process of learning in its true sense.

"Maintaining Sanity in the Classroom" - Dreikurs, Grunwald, Pepper

DISCIPLINE

I. Some general principles

A. Teacher attitudes and understanding

1. Have a sincere regard and concern for students

2. Be sensitive to the needs of all students

3. Discern the ways in which children differ

II. Preventive Techniques

A. Ways of Maintaining an Orderly Classroom

1. Make a good beginning. i.e., when meeting a class for the first time, the teacher should explain his procedures, standards, and regulations, and the reasons for them. As a rule of thumb, it is better to be relatively strict with new groups, but the requirements and regulations set by the teacher should be reasonable. It is much less difficult to ease requirements than to stiffen them.

2. Maintain standards. Once the teacher has adopted certain standards and procedures, he should insist that they be observed. To make an exception is to invite another exception, and another, and another.

3. Do not begin a class until pupils are attentive. Until orderly conditions are present, effective learning cannot take place. Stop when there is noise/inattention.

4. Be thoroughly prepared every day. Thorough preparation assures the teacher of knowing, during every minute of the class period, exactly what he will do and how he will do it. Such a knowledge by adding to his self-confidence, enables him to unfold the day's activities with assurance. Students are unlikely to probe for weaknesses in the teacher if he appears to be a competent individual. In additional, thorough preparation keeps the work of the day moving along in such a way as to keep all students continuously active, leaving no time for mischief.

5. Vary your procedures. Variety adds spice to teaching, whereas the same routine day after day is flavorless. It is difficult to hold the attention of pupils for long periods of time with an inflexible method of teaching. Variety of procedures adds interest, and helps to maintain attention. A common mistake made by teachers is to overuse the lecture method. They explain, describe, or narrate to the students, while their pupils occupy themselves by taking notes -- a procedure which, if continued, is deadly to holding interest and attention.

6. Keep your students active and they will have no time for mischief. The teacher's lesson plan should include ways of keeping all students active at all times, through written work, oral work, or board work. When a topic is being explained or discussed, the teacher may call on a pupil to summarize or the pupil may be asked a question on the material. The entire class should be thinking about the teacher's question and each pupil's answer each time a question is asked.

7. Treat your pupils as individuals. Know your students by name as soon as possible. Even though it is well for a teacher to be friendly toward his students and to show an interest in them, he should also keep in mind that there should be a line of demarcation between pupils and their teachers beyond which neither should step. This is not to say that the teacher should feel superior to his pupils; it is to say, rather, that a teacher should maintain his dignity as one in authority or he will lose the respect of his pupils and be unable to control them. Try not to be influenced by the reputation a pupil has earned in other classes with other teachers. Take your students at face value.

8. Be fair. Although a teacher is sure to like some of his students better than others, he should never allow this feeling to enter into the evaluation of their progress. It is sometimes necessary to consider extenuating circumstances, or to sprinkle justice with charity, but this acceptable practice is altogether different from allowing feelings to influence judgment.

9. Admit mistakes

10. Have a sense of humor within limits.

III. Ways of Dealing with Disorderly Pupils

A. Handling group infractions or misbehavior.

1. Don't wait until a class is out of hand.

2. Focus on individual culprits rather than upon the class in general. Divert ringleaders by questions or assigned appropriate activities.

3. Keep your temper and your poise. Pupils may be testing you.

4. Avoid threats you cannot or will not carry out.

5. If difficulties continue, analyze possible causes: your manner or attitude; management of routines, teaching procedures, etc.

6. Seek help or advice from experienced colleagues, dean or supervisor.

B. Handling Individual Offenders in the Classroom

1. Minor infractions (Stop and look at student; talk to student quietly; change seat). In a private talk, the pupil is more likely to cooperate,

there being no need for him to resort to face-saving or attention-getting devices, as there would be in the presence of his classmates. The danger in changing a student's seat is that the source of trouble may merely be transferred from one room location to another, in which event the affected student may need to be seated close to the teacher. More often than not, however, the student whose seat has been changed, although perhaps tending to act up for a day or so to save face by temporarily defying authority, will soon settle down in the new location. And even if the result is unsatisfactory, at least the teacher will have experimented with a less offensive approach before adopting a severer one.

2. After a correction, treat the issue as closed.

3. Emphasize the behavior, not the pupil.

4. Don't argue; avoid open clashes.

5. Don't force an issue so pupil is made to lose face; don't humiliate.

6. Frequent questioning. A measure that sometimes produces quick results with a pupil who is talkative or inattentive is to direct questions to him more frequently than to other students. It will not take the student long to recognize that he is questioned more frequently when he is disorderly than when he is not, and usually he will begin to give more attention than he did previously.

7. Don't force a student to publicly apologize to the class. A public apology usually accomplishes nothing constructive from the viewpoint of teacher-pupil relationships, and the smoldering resentment the student carries within him certainly is not conducive to willing work on his part.

8. Other negative characteristics to be avoided in the meeting out of punishment is anger, frequent warnings, threats, scoldings, and sarcasm. The teacher should deliver the needed reprimand in a normal voice, with a minimum of emotion and in a non-judgmental way. Even though sarcasm admittedly is a function of time, person and place, a studied avoidance of the very appearance of it in the classroom is a safer approach.

C. Dealing with discipline problems that cannot be settled in a class.

1. Talk to offender in private. Determine causes of misbehavior and try to reach agreement or understanding.

2. Make the punishment fit the offense.

3. Leave offender with the feeling that he has a chance to redeem himself.

4. Use the resources at your disposal: check pupil's permanent record, get suggestions from teachers who know pupil, speak with pupil, analyze

your approaches, communicate with or see parent in accord with school procedures, channel his energies into useful approaches, keep an anecdotal record for chronic offenders, consult informally with supervisor, counselor or experienced colleague.

5. After exhausting your resources without results:

   a. Refer case to supervisor, dean or counselor in accordance with school regulations.

   b. Include on referral form the specific offense and measures you have taken to help the offender.

   c. Send additional material such as anecdotal record.

IV. **How can I get pupil's to work with me not because they have to but because they want to?**

A. Show an individual interest in your pupils -- their health, their problems, their achievements.

B. Establish class procedures and standards of behavior.

C. Trust pupils without being an easy mark. Take a pupil's word unless you have a good reason not to.

D. Give pupils a feeling of success through assignment of reasonable tasks and your personal encouragement.

E. Praise the class and individual pupils for good work, cooperation or improvement. Show that you appreciate their efforts and abilities.

F. Stress the positive -- what should be done rather than what should not be done.

G. Be fair and consistent. Don't accept infractions at one time and condem them at another time.

H. Show pupils the same respect and courtesy that you expect from them. Avoid sarcasm. Don't be arrogant or condescending.

I. Show your pupils that you like them and are concerned about them. They will respond in kind.

HOW SHOULD ONE HANDLE SPECIFIC TYPES OF DISCIPLINARY INFRACTIONS?

CONSTANT TALKING: Ask pupil questions so that he myst pay attention.
Check pupil's work.
Assign to activity such as chalkboard
Change his seat.

CALLING OUT OR INTERRUPTING:	Insist that pupils must raise their hands and then be called on to recite. Refuse to accept or give credit for an answer that is called out. Stress that other pupils should be given a chance to respond. Talk to pupil after class if necessary.
REFUSAL TO DO WORK OR OPEN DEFIANCE:	Make notation, then speak to pupil privately. Determine cause: work not understood, physical indisposition, emotional reaction, lack of motivation, etc. Suit follow-up action to cause. Use resources as needed: Checking pupil's record for his background and achievement, etc.
FOOLING AROUND OR HORSEPLAY:	Keep class working so that the offender or offenders are isolated. Insist calmly but firmly that nonsense be stopped. Check work and make notation in marking book or a Delaney card. If misbehavior stops quickly, continue lesson as if nothing has happened. If misbehavior continues or is repeated, see offenders after class. Follow up as needed.
IMPROPER LANGUAGE:	Indicate calmly that language is improper and uncalled for. Don' t take obsenity as personal affront but as offense against class and school. If remark is just one outburst of temper, keep your own record but try to adjust matter by talking to pupil in private. Don't insist on public apology, if pupil is repentant, his attitude will be evident to the class. If improper language is repeated or is part of a poor behavior pattern, send referral form to dean.
LEAVING ROOM WITHOUT PERMISSION:	Don't try to stop pupil by physical force. Send another pupil with note to inform dean. Mark pupil as cutting for time he is out, and follow school procedure for cutting. Continue your lesson. Follow up when pupil returns, using resources at your disposal. Suit your actions to the causes and the pupil's attitude.

ARGUMENT BETWEEN PUPILS:	Separate pupils quickly. Seat them in different parts of the room. If argument stops, see each pupil separately. Make sure argument has been settled. If argument threatens to continue after class or after school, notify dean's office. If argument continues in class, send one of the disputants out with another pupil, to the dean, with a note. If fight starts, send for assistance and try to break up fight with help of other pupils. Don't step in between the combatants unless they are younger or smaller children.
TEACHER OR PUPIL POSSESSIONS MISSING:	Give culprit chance to return article immediately without penaly, and possibly without detection. If article is not returned and you find culprit, send referral form at once. Theft, as distinct from mere mischief, is a serious offense. If article is not returned or discovered, send promptly for dean. Keep class in room until dean or security arrives.

Class Control, Class Discipline and Their Relation to Good Teaching

I. Causes of Trouble with Class Control and Discipline

A. Starting to teach before the entire class is in order and "under control."

B. Poor Teaching Techniques: direct correlation between teaching and class control and problems of discipline.

1. Weak and ineffective motivation; lack of interest in the lesson.
2. Weak or uncertain aim of lesson.
3. Poor questioning; failure to challenge or to stimulate pupils.
4. Inadequate use of the blackboard and other visual aids.
5. Not enough variety of procedure from day to day.

C. Teacher's personality uncertain and variable from day to day; not animated, dynamic or forceful, inconsistent enforcement of rules.

D. Teacher lacks adequate knowledge of the psychology of adolescents.

1. Teacher "sees" or "hears" too much.
2. Teacher loses temper and insults or humiliates pupils in front of the class.
3. Strictness not tempered by a sense of humor.
4. Teacher "not consistent" in day-to-day approach to class.

E. Teachers often direct attention to front of the room and part of the class, rather than directing himself to the entire class.

II. Constructive Suggestions for Better Class Control and Better Discipline

A. Make conscious effort to improve rapport; be firm but fair; don't humiliate or belittle students' efforts. Develop a sense of humor but do not attempt to compete with top-flight comedians; be on friendly terms with the students; make yourself available for post-class, brief discussions; don't "see" or "hear" everything; teach with spirit and animation; try to gain respect and good will of your pupils; do not teach "down" to the students.

B. Start teaching only when all pupils are seated, in order, and under your complete control; do not tolerate pupils coming late to class; have students remove gum and place in wrapper; permit no wearing of hats in class; permit no eating of food or drinking of soda in class; permit no student to "rest his head" on desk or "sleep" during your teaching recitation.

C. Careful and Well-Thought-Out Lesson Planning; provide for --

1. Stimulating motivation that will excite interest in the lesson and give a sense of importance.

2. 5 or 6 pivotal questions that will call for meaningful responses, provoke discussion, give opportunity for pupil curiousity.

3. Colorful illustrative material and other visual aids.

4. Pupil activity and live pupil participation.

D. Read some up-to-date books on adolescent psychology. Guard against offending pupils by personal remarks; respect the dignity of each individual pupil.

E. Direct your attention to the entire class; keep the ENTIRE class under your control every moment of the period. Draw pupils from every part of the room into active participation.

F. See to it that every day you dismiss the class. Keep teaching to the last moment.

III. **Dealing With Cases of Discipline**

A. First rule: don't make a threat you won't or can't carry through.

B. Avoid "cross the room" argument with a pupil.

C. Control yourself; ask pupil to see you after class - it is most important to avoid giving pupil an "audience situation" which encourages showing off and open defiance of the teacher; you must see the pupil privately.

D. Change seats of trouble-makers, without being too conspicuous about it.

E. In serious cases take up the issue with the Chairperson man and the Deans.

HOW CAN WE IMPROVE SUBJECT CLASS ATTENDANCE?

CAUSES: **What are the reasons that students do not come to class?**

1. They are ill.
2. They have special problems at home (parent ill, special responsibilities for a young child).
3. They are misprogrammed.
4. They would prefer to use the time to earn money.
5. They have no interest in the subject.
6. They have no hope that they will pass (perhaps they have failed math many times before).
7. They would rather spend the time with their friends (they can see people they know hanging out right outside the school).
8. They are prone to math anxiety and feel frightened.
9. They are frustrated.
10. They are bored.
11. They would rather attend special school events (rally, athletic event, trip, band concert, another class).
12. They feel it is more important to do something else (often related to school life: replace lost transit pass; apply for working papers; speak to guidance counselor) at the time the subject class meets.
13. The room is too hot and/or too ugly.
14. There is no place for them to sit.
15. They do not see the course as relevant to their lives.
16. They believe they know it already or can learn it on their own (especially if repeating the course).
17. They are uncomfortable facing the teacher's authority; they are used to taking care of themselves and want to make their own decisions; they feel the teacher treats them like children rather than the adults they feel themselves to be. (This happens especially when parents are absent from the home or disinterested and/or when the teacher has very rigid rules about performance and behavior.)
18. They have fallen into the grip of the habit of cutting; although they want to keep coming, they find it hard to change their ways.

HOW CAN WE IMPROVE SUBJECT CLASS ATTENDANCE?

What can the subject teacher do to improve attendance?

1. Explain why it is important to come. (What will the student gain?)
2. In each lesson, integrate the math in the students' lives in a relevant way.
3. Be sure that the student is correctly programmed.
4. Speak to the students as they arrive. Welcome them to class.
5. Smile.
6. Create the best environment possible. (Be polite; decorate; open windows)
7. Give each student personal attention.
8. Show that you notice (and care about) what the student is doing; praise good work; provide support when the student has trouble.
9. Ask the student to help you in a special way (you are counting on him to erase the board, explain HW, etc.)
10. Call the parent and discuss the student's progress.
11. Share your long range plans with the student ("Next week, we will use what we're doing today to...").
12. Avoid words like "check" or "mark" when "examine" or "review" is more appropriate (to defuse anxiety).
13. BE ENTHUSIASTIC.
14. Plan lessons which are stimulating and interesting.
15. Explain where extra help is available or arrange to provide it yourself.
16. Give recognition for perfect or near-perfect attendance (praise, certificate, badge, button, prize, cash, party). This may be a department-wide effort. For some courses, it may be best to offer a reward for a marking period's good attendance; for others, a reward for a term's effort would be more suitable.

REDUCING MATHEMATICS ANXIETY

Learning anxieties, born of previous defeats and frustrations, impair the learning ability of many students. Of all these learning anxieties, math anxiety is the most widespread and most disabling. While math anxiety infects some average and even above average students it is epidemic and paralyzing among retarded students. Prescriptions that ignore it are unresponsive to an aspect of the diagnostic process that demands attention.

I. **Causes of Math Anxiety**

- A. Previous personal experiences with parents, siblings, et al.
- B. Sex-role stereotypes, social pressure.
- C. The lingering effects of poor schooling and irregular attendance.
- D. Attitude towards authority.
- E. Student perception that math content is inflexible or the assumption that assigning symbols and making abstractions is depersonalized.

II. **Symptoms of Math Anxiety**

- A. Sweaty palms; flushed cheeks.
- B. Inability to hear or concentrate on directions.
- C. Fear of asking questions, of revealing "ignorance".
- D. Inattention and avoidance.
- E. Withdrawal and inactivity.
- F. Detachment which makes it impossible to function (math block).

III. **Organizational Practices**

- A. Reviewing departmental class organizational practice.
  1. Insuring proper class placement of students.
  2. Limiting the ability range of students.
- B. Designing varied grouping practices within the classroom: individual, small group, class.

IV. **Creating a Responsive Environment**

- A. Teacher-student relationships.
  1. Warmth and responsiveness, optimism and energy
  2. Firmness without threats or criticism
  3. Commitment to student growth
  4. Overt encouragement and praise
  5. Knowing students personally: using questionnaire and similar devices
  6. Showing that math takes effort (even by the teacher)
  7. Discussing math anxiety openly
  8. Inviting students' reaction to the level of difficulty of work and of the text
  9. Avoiding emphasis on covering a predetermined amount of material

B. Classroom Climate and Tone

1. Uncluttered appearance
2. Informal seating - relaxed atmosphere
3. Attractive decoration: plants, pictures, etc.
4. Varied and abundant materials
5. A quiet corner for concentrated efforts, mathematical browsing
6. Provision for display of students' work
7. Freedom to ask questions, to admit lack of knowledge

V. Instructional Procedures

A. Diagnosis and Prescriptions

1. Individual diagnosis
2. Interpreting student error patterns
3. Individual prescriptions

B. Performance Objectives

1. Teaching for understanding
2. Learning how to solve problems

   a. Focusing on realistic expectation
   b. Emphasizing students' mathematical capabilities
   c. Removing the pressure required for memorization by accepting the students' use of reference material, use of texts for finding formulas, etc.
   d. Stressing that the approach is more important than the answer

Dangers to avoid: Meaningless memorization, excessive drill and practice, labeling problems or math as easy, stress on speed, etc.

C. Motivation

1. Using current needs
2. Using possible careers

D. Development

1. Stressing students' discovering for themselves
2. Developing high but realistic expectations
3. Providing a variety of activities to meet individual learning styles
4. Clarification of language problems

   a. Technical words frequently misunderstood: ratio, fraction, least common denominator, etc.
   b. Common words frequently misunderstood: multiply, of, subtract, etc.
   c. Mathematical terms with multiple meanings: negative, minus, etc.
   d. Problems underlying symbols like -, X, $(\ )^2$

E. Materials of Instruction

1. Using practical real-life materials
2. Using varied materials for stimulation and to meet individual needs
3. Training students to use texts as reference
4. Avoiding "over-simplified" materials

F. Socialization

1. Encouraging verbal analysis of problems
2. Encouraging students to explain to one another
3. Forming small groups (2-6) for constructive interaction

VI. Setting Standards

1. Developing high but realistic expectations
2. Developing standards that are individual, not competitive
3. Insuring consistent recordkeeping
4. Providing feedback to students and parents on progress

VII. Taking Tests

1. Discussing math anxiety
2. Skimming the entire test
3. Starting with a problem students can solve easily
4. Encouraging students to defer more difficult problems and return to them after other problems are solved
5. Asking questions if the wording is vague or unclear
6. Searching for solution, not formulas
7. Training students to own personal learning style
8. Remembering that tests are not the last word in determining self-worth

Some DO's and DON'Ts — Meeting with the Parent

1. Try to arrange the conference so that there will be no interruptions.

2. The teacher's greeting should be warm and friendly, with a warm up topic to open the conversation.

3. Begin with a positive statement of the student's abilities. The parent is less defensive, and it conveys interest and liking for the student.

4. Share observations about the student, comparing responses at home and at school before making an interpretation or judgment.

5. Let any advice grow out of mutual discussion and a growing insight on the part of the parent into the reasons for the behavior.

6. Discover how the parent is thinking and feeling about the student. The teacher cannot understand the student's behavior until he/she knows the parent's attitude.

7. Listen to the parent's complaints or criticisms of the school, accepting them as evidence of interest or as suggestions to be considered.

8. If a parent gives his/her reason for the student's behavior, accept it, and, if possible, lead the discussion on to other possible causes. Behavior is a result of many causative factors.

9. Listen, and then listen some more. The teacher did not invite the parent to lecture but to obtain as well as give help. Encourage the parent to talk, and listen to what is said.

10. Try to close the conference with a constructive, hopeful statement.

11. It is easier to build a cooperative relationship if the teacher is not behind a desk. (Use a table if available.)

12. The educator should not argue with a parent. Arguing will arouse resentment.

13. The teacher should not try to push his/her thinking onto a parent. Conclusions and recommendations should be reached through a process of discussion and mutual thinking.

14. The teacher should not criticize harshly or negatively. Destructive criticism can be fatal to build a cooperative relationship in parent-teacher conferences since most parents cannot be objective about their children. The emotional involvement is too great.

15. The educator should not give direct advice unless it is requested, and never as the single successful solution.

16. Do not imply that the parent or home is to blame for the student's behavior.

17. Do not overlook influences other than the home on the student's life.

18. Avoid discussing other children in the school. Such comments direct attention from the problem at hand, encourage competitiveness, and often harm school and neighborhood relationships.

19. Do not use blanket words such as immaturity and insecurity.

20. Do not assume that the parents want help or advice. Such assumption usually brings resistance because it implies a form of criticism.

21. Do not criticize or blame past school experiences or teachers because such comments tend to destroy confidence in all education. Attention should be centered on present needs and on plans for the future, not on past mistakes.

Data Collection Sheet for Simulation of Statistics Lesson

Name (Last, First) _____ School _____

Eye color _____ Hair color _____ Height (inches) _____

Age _____ Weight _____

- How did you feel about being asked for your age and weight?
- For the purposes of a lesson on statistics, what other data could the instructor have ased for that would have served just as well?
- What implications does this simulation have for the classroom teacher?
- How can the teacher anticipate the effects of classroom interactions on the feelings of students?

Situations for Discussion by New Teachers: Classroom Management

1. A high-achieving student finished the regular class assignment ahead of schedule and is now sitting at this seat reading a book.

2. One student comes habitually late to class every day. When you confront the student, the student says that the Phys Ed teacher keeps the students late everyday. What do you do?

I. Logic

A. Truth Value of Simple Sentences

1. Closed sentences
2. Open sentences
   a. Replacement set
   b. Solution set
3. Negations

B. Truth Value of Compound Sentences

1. Conjunction
2. Disjunction
3. Conditional
4. Related conditionals
   a. converse
   b. inverse
   c. contrapositive
5. Biconditional

C. Tautologies

1. Proof using truth tables
2. Applications

II. Algebra

A. Operations on Algebraic Expressions

1. Review of operations with signed numbers
2. Use of variables
   a. Order of operations
   b. Evaluating algebraic expressions
3. Addition of polynomials
   a. Properties
   b. Combining like terms
   c. Combining fractions with like denominators
4. Multiplication of polynomials
   a. Powers ($x^a \cdot x^b = x^{a+b}$)
   b. Products using monomials
   c. Product of binomials
   d. Equivalent fractions
   e. Combining fractions with unlike denominators
   f. Multiplication of fractions
5. Simplification of algebraic expressions using addition and multiplication

6. Division of polynomials by monomials
    a. Powers ($x^a \div x^b = x^{a-b}$)
        1. Zero and negative exponents
        2. Scientific notation
    b. Simplification of fractions
    c. Division of fractions
7. Factoring
    a. Prime factorization
    b. Common monomial
    c. Binomial factors of a trinomial
    d. Binomial factors of the difference of two squares

B. Solution of Equations and Inequalities

1. Open sentences
    a. Solution set
    b. Representation of solution set on number line
2. Techniques for solving linear equations and inequalities
3. Applications
4. Techniques for solving quadratic equations
    a. Zero property of multiplication
    b. Solution by factoring
    c. Solution by quadratic formula (optional)

C. Ratio and Proportion

1. Definition
    a. Ratio
    b. Proportion
    c. Means and extremes
    d. Percent as a ratio
2. Solution of proportions
3. Applications
    a. Percent
    b. Similar figures
    c. Direct variation

D. Real Numbers

1. Rational and irrational numbers
2. Examples of irrational numbers
    a. Non-repeating decimals
    b. Irrational roots
    c.
3. Rational approximation of irrational numbers
4. Operations with radicals
    a. Simplification
    b. Multiplication and division
    c. Addition and subtraction
5. Applications
    a. Solution of incomplete quadratic equations
    b. Problems involving Pythagorean Theorem
    c. Use of quadratic formula (optional)

III. Geometry

A. Undefined Terms--Point, Line, and Plane

B. Review of Geometric Figures

1. Parallel and intersecting lines
2. Angles
    a. Degree measure
    b. Right, acute, obtuse, straight
    c. Supplementary and complementary
    d. Vertical
    e. Alternate interior, corresponding
3. Perpendicular lines
4. Simple closed curves
    a. Polygons
    b. Circle

C. Triangles

1. Classification
    a. Scalene, isosceles, equilateral
    b. Acute, right, obtuse
2. Sum of the measures of the angles of a triangle
3. Exterior angle of a triangle
4. Base angles of an isosceles triangle
5. Comparison of triangles
    a. Congruence
    b. Similarity
6. Pythagorean Theorem
7. Application to Problem Solving

D. Quadrilaterials

1. Classification and properties
    a. Parallelogram
    b. Rectangle
    c. Rhombus
    d. Square
    e. Trapezoid
2. Sum of the measures of the angles

E. Polygons

1. Review of polygons
2. Perimeter
3. Area
4. Similar polygons
    a. Ratio of perimeters
    b. Ratio of areas

F. Circle

1. Circumference
2. Area

G. Transformations

1. Intuitive notions
   a. Line reflection
   b. Translation
   c. Rotation
   d. Dilation
2. Congruence and similarity
3. Symmetry
   a. Line
   b. Point

H. Solids

1. Classification
   a. Prism
   b. Rectangular solid
   c. Pyramid
   d. Right circular cylinder
   e. Cone
   f. Sphere
2. Volume
3. Comparison of volumes of similar solids

IV. Analytic Geometry

A. Review of Rectangular Coordinate System

B. Linear Relations and Slope

1. Graphing linear relations
2. Slope of a line
3. Slope intercept form

C. Direct Variation

D. Graphing Linear Inequalities

E. Graphic Solution of Systems of Linear Equations

F. Algebraic Solution of Systems of Linear Equations

1. Substitution method
2. Addition-subtraction method

G. Graphic Solution of Systems of Linear Inequalities

H. Area Using Coordinates

I. Optional Graphing

1. $y = |x|$
2. $|y| = |x|$
3. $|x| + |y| = c$
4. $y = ax^2$
5. $y = ax^2 + b$

V. Probability

A. Review of Terms

B. Probability of Single Events

1. Theoretical vs. empirical probability
2. Simple probability and notation
3. Problems involving AND
4. Problems involving OR
5. Probability of the complement of an event

C. Counting Procedures

1. Sample space
   a. List of ordered pairs or n-tuples
   b. Tree diagrams
   c. Dot graphs
2. Counting Principle
3. Permutations
   a. Factorial notation
   b. ${}_nP_n$
   c. ${}_nP_r$

D. Probability of Events Occuring Jointly

1. Extension of Counting Principle

VI. Statistics

A. Collecting and Organizing Data

1. Sampling or census
2. Tally, chart, frequency table
3. Frequency histogram
4. Cumulative frequency histogram

B. Measures of Central Tendency

1. Mean
2. Median
3. Mode

C. Other Significant Measures

1. Quartiles
2. Percentiles

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COURSE III (from State Education Dept.)

Scope and Content

I. Complex Numbers (recommended time: 25 days)

A. Further study of the real number system

B. Extension of algebraic techniques

C. Introduction of the complex number system

D. Quadratics with complex roots

E. Absolute value: equations and inequalities

II. Relations and Functions (recommended time: 25 days)

A. Definitions and properties

B. Exponential and logarithmic functions

C. Common logarithms and computation

III. Circular Functions (recommended time: 60 days)

A. Further work with the geometry of the circle

B. Circular functions and their graphs

C. Trigonometric identities and equations

D. Applications to area, Law of Sines, Law of Cosines, right triangle trigonometry

IV. Transformation Geometry (recommended time: 20 days)

A. Review of mapping

1. Definitions, one-to-one, identity mappings, inverse mappings
2. Some geometric examples

B. Line Reflections

1. Considered as mappings
2. Properties "preserved" by line reflections
3. Line symmetry
4. Coordinates of image of point (x, y) under reflection in special lines of coordinate plane

VI. Optional Topics

A. Polar coordinates

B. Number theory

C. Mathematical induction

D. Sequences and series

E. Matrices

F. Synthetic substitution, remainder theorem, and factor theorem

Outline of Fundamentals of Mathematics Volume 1

UNIT I - Formulas

The following is a summary of the aims and performance objectives for each of the lessons in Volume 1 of the FM curriculum guide.

1. To use symbols to represent numbers and operations

   - state the operation associated with the words: sum, product, difference, quotient, more than, less than, diminished by, increased by, factor, etc.
   - translate an English phrase expressing a binary operation into numerical symbols
   - use a variable in a binary expression
   - represent a product of two or more numbers in the following four ways: X, • , ( ), and no sign at all.
   - evaluate numerical expressions containing only one binary operation where the numbers are whole numbers

2. To evaluate algebraic expressions with one or more variables

   - translate a phrase containing one or more variables into an algebraic expression.
   - replace variables in an algebraic expression with their designated numerical values.
   - change the format of a numerical expression for computational purposes.
   - perform the indicated operations in numerical expressions.
   - differentiate between algebraic and numerical expressions.

3. To evaluate expressions using the correct order of operations

   - state the order in which operations are to be performed
   - evaluate numerical expressions using the correct order of operations
   - evaluate algebraic expressions for stated values of the variables, using the correct order of operations
   - state that a numerical expression has only one value

4. To use parentheses as a grouping symbol

   - recognize that operations within parentheses are to be performed first.
   - evaluate numerical expressions with parentheses.
   - recognize the "distributive property of multiplication over addition"

5. To evaluate numerical expressions containing exponents

   - express a number written in exponential form as a repeated multiplication.
   - state the meaning of base, exponent and factor.
   - evaluate numerical expressions containing exponents.
   - compute the squares and cubes of numbers.

6. To evaluate algebraic expressions containing exponents

   - evaluate algebraic expressions containing exponents.

   - rewrite variables with exponents as repeated multiplications.

7. To evaluate the subject of formulas

   - state a need for using formulas

   - state the difference between a formula and an expression

   - recognize the subject of a formula

   - find the value of the subject of a formula

UNIT II - Introduction to Algebra

1. To use symbols of equality and inequality

   - compare two whole numbers verbally using the words "is greater than", is less than," "or is equal to."

   - use symbols of equality and inequality to compare two whole numbers.

   - recognize whether a sentence using $<$, $=$, or $>$ is true or false.

2. To order numbers on a number line

   - construct a number line using a unit length with end points at 0 and 1, and extending to include all whole numbers.

   - locate any positive whole number on the number line; thereby, showing understanding of the one-to-one correspondence between whole numbers and points on the line.

   - compare the positions of two points on the line to their relative size (smaller is to the left).

   - describe the order of two numbers on the number line using $<$ and $>$.

   - describe the order of two lettered points on the line using $<$ and $>$.

   - state the rule, "if a is to the right of b, then $a > b$ or $b < a$.

   - understand the correspondence of rational numbers to points on the number line by subdividing a unit on the number line (quarter units only).

3. To introduce ruler fractions

   - recognize when two rational numbers are equivalent (denominators 2,4,8,16).

   - use $<$ or $>$ to describe the order of ruler fractions.

   - change an improper fraction into a mixed number and vice versa.

4. To read a ruler

   - read the markings on a ruler.

   - measure line segments with a ruler.

   - name line segments using two capital letters.

   - compare lengths of line segments.

5. To comprehend the meaning of directed numbers

   - represent gains and losses, right and left, above and below zero, etc. as signed numbers.

   - write the opposite of a given situation.

   - write the opposite of a given signed number.

6. To extend the number line to include negative numbers

   - draw the number line to the left of 0 to allow for movement in both directions.

   - rename the points on the number line using positive and negative numbers.

   - define movement to the right on a number line as positive and movement to the left as negative.

   - express the distance between 2 points as a signed number.

7. To establish an ordering property of positive and negative rational numbers

   - express the order of two positive integers in terms of their position on the number line (a is to the left of b, so $a < b$).

   - use the ordering symbols for all signed numbers (i.e., $-2 > -6$ because -2 is to the right of -6).

   - find the least common denominator, given two or more fractions where the denominator of one is a multiple of the denominator of the other.

8. To compare two rational numbers

   - find the least common denominator, given two fractions with relatively prime denominators.

   - find the least common denominator, given two fractions with unlike denominators which are not relatively prime.

   - use $<$, $>$. and $=$ to compare rational numbers with unlike denominators.

9. To compare more than two rational numbers

   - find the LCD for more than 2 fractions with unlike denominators

   - order more than 2 rational numbers with unlike denominators

10. To add signed numbers using the number line

- add two whole numbers on a number line.
- add positive and negative numbers on the number line. (Sums may be either positive or negative.)
- state and demonstrate the commutative property of addition on the number line.

11. To develop a rule for the addition of signed numbers

- state and apply the rules for adding signed numbers.
- define absolute value of an integer as its distance from 0 on a number line.
- define additive inverses as two numbers whose sum is 0.
- find the additive inverse of a number.

12. To add rational numbers with like signs

- add rational numbers with like denominators and like signs.
- add rational numbers with unlike denominators and like signs

13. To add rational numbers with unlike signs

- add rational numbers with like denominators with unlike signs
- add rational numbers with unlike denominators with unlike signs
- subtract rational numbers

UNIT III - Metric Measurement

1. To introduce the units of linear measurement in the metric system

- identify the meter as the basic unit of linear measure in the metric system
- express the numerical relationships between the meter and the millimeter, the centimeter, and the kilometer.
- recognize (visually) the similarity of the meter and the yard without making conversions.
- recognize the relative sizes of and indicate appropriate uses for the millimeter, centimeter and kilometer.

2. To read and interpret large numbers and decimals

- write a numeral given in verbal form
- name the place value of any digit in a whole number up to the millions place
- name the place value of any digit in a decimal number up to the ten thousandths place
- read any number expressed in symbol form from .0001 to 9,999,999 in words and vice versa

3. To develop understanding of the metric ruler and the relationship between the meter and the centimeter

   - identify and read the centimeter markings on a metric ruler
   - express the subdivisions of a centimeter as tenths of a centimeter
   - measure lengths in meters and centimeters with a ruler
   - apply the rule for converting meters to centimeters by multiplying by 100

4. To develop understanding of the relationship between the centimeter and the millimeter, and between the meter and the millimeter.

   - identify and read the millimeter markings on a ruler
   - convert measurements from cm to mm and m to mm (multiply by 10 and 1000).
   - measure lengths in millimeters

5. To convert from smaller to larger metric units

   - convert from millimeters to centimeters, from centimeters to meters and from millimeters to meters.

6. To develop understanding of the relationship between the meter and the kilometer

   - compare the relative sizes of kilometers and miles, e.g., 50 miles is about 80 kilometers.
   - convert measurements from meters to kilometers and vice versa

7. To provide practice in converting from one metric unit of linear measure to another, with a variety of application exercises

   - apply the metric lengths to various problem situations
   - express the quotient of two whole numbers as a decimal

8. To introduce the units of measure of weight in the metric system

   - state that the gram is the basic unit of weight in the metric system
   - state that there are "about 30 grams to one ounce" (no conversions from one system to another are to be made)
   - express the relationships among grams, milligrams and kilograms
   - compare the relative sizes of, and indicate appropriate uses for, the milligram, gram, and kilogram

9. To further develop understanding of the relationship between the gram and the kilogram

   - convert from grams to kilograms and vice versa.

10. To develop further understanding of the relationship between the gram and the milligram

    - convert from grams to milligrams and vice versa.

UNIT IV - Plane Geometry

1. To identify triangles and to classify them according to the lengths of their sides

    - illustrate the importance of geometric shapes by referring to their uses in architecture and elsewhere.

    - state the definition of triangle.

    - divide the set of triangles into three groups: scalene, isosceles, and equilateral, relating the classification to the number of equal sides.

2. To define "angle" and types of angles and to measure and label angles

    - describe an angle as a rotation (as the angle between the hands of a clock).

    - state that one complete rotation is divided into 360 units called degrees.

    - state that one-quarter of a rotation is a right angle (90°).

    - state that one-half of a rotation is a straight angle (180°).

    - state a definition for an acute angle a right angle, an obtuse angle, and a straight angle.

    - select by sight from given diagrams an acute right obtuse and straight angle.

    - to find the measure of an angle using a protractor.

3. To determine the sum of the measures of the angles of a triangle

    - state the relationship that the sum of the measures of the angles of a triangle is 180

    - apply this relationship to find the measure of the third angle of a triangle when the measures of two angles are given.

    - state that a triangle cannot include two right angles, two obtuse angles or a right angle and an obtuse angle.

4. To classify triangles according to the measures of their angles

    - determine when a triangle is acute, right, or obtuse.

    - state that equilateral triangles are equiangular.

    - state that an isosceles triangle has two angles of equal measure.

    - name the parts of a right triangle.

    - state that the hypotenuse is the longest side of a right triangle and is opposite the right angle.

    - state that the legs of a right triangle form the right angle.

5. To study the Pythagorean relationship

   - state that a and b in the Pythagorean relationship ($a^2 + b^2 = c^2$) are the legs, and c is the hypotenuse.
   - make a list of squares of whole numbers from 1 to 20.
   - express a perfect square as the second power of one of its factors.
   - state that a perfect square can be written as the product of two equal factors.
   - determine if a triangle is a right triangle using the Pythagorean relationship.

6. To find square roots of perfect squares and apply the Pythagorean relationship

   - read the symbol $\sqrt{\ }$ as "the square root of"
   - state that the square root of a number is one of its two equal factors.
   - find the square roots of perfect squares.
   - use the Pythagorean relationship and the concept of finding a square root (by trial and error) to find the hypotenuse of several right triangles with whole numbered sides.
   - find the length of a leg of a right triangle using the Pythagorean relationship.

7. To identify quadrilaterals and to distinguish among the various special quadrilaterals, including the parallelogram, rectangle, rhombus, square, and trapezoid

   - define quadrilateral.
   - recognize the "special" quadrilaterals by their shapes.
   - use the terms equilateral and equiangular when referring to the appropriate quadrilaterals.
   - recognize the differences (and similarities) among the various types of quadrilaterals, emphasizing parallel lines, lengths of sides, and right angles.
   - know some of the properties of the diagonals in the quadrilaterals (esp.: bisecting each other, equal lengths, and forming right angles).

8. To identify polygons of more than four sides

   - name polygons containing 5, 6, 8, 10, and 12 sides.
   - define "regular polygon."
   - state the relationship among the angles of some polygons that allows them to tessellate or "fit" together. (The sum of the measures of the angles about a point + 360°.)
   - state a relationship between the number of sides and the number of diagonals in a polygon.
   - state a relationship between the number of sides in a polygon and the number of triangles with a common vertex that may be found in the polygon.

9. To identify and discuss some of the points and lines in a circle

   - define a circle as the figure formed by all the points a given distance from a fixed point.
   - state that a circle can be approximated by constructing a regular polygon with a "large" number of sides (24 sides, for example).
   - define and draw a radius, a diameter, a chord, an arc, a sector.
   - state the relationship between a radius and a diameter.
   - solve arithmetic problems relating diameters and radii, which use fractions and decimals.
   - construct a circle using a compass.

10. To calculate the perimeter of any polygon

   - define perimeter.
   - use fractions and decimal numbers to calculate perimeters of polygons.

11. To solve various application problems involving perimeters of polygons

   - use some properties of triangles (isosceles, equilateral) and polygons when solving perimeter problems.
   - find the length of a missing side of a polygon when the perimeter and the other sides are given.
   - perform subtractions with mixed numbers.

12. To calculate the circumference of a circle using $\pi = 3.14$

   - define circumference.
   - measure circumferences of common objects using string and a ruler, or a tape measure.
   - find an approximate value of π by relating the circumference and the diameter of objects.
   - find the circumferences of circles using 3.14 for π, where the radii and diameters are decimals or whole numbers.

13. To calculate the circumference of a circle using $\pi = \frac{22}{7}$

   - find the circumference of circles using $\frac{22}{7}$ for π where the radii and diameters may contain fractions.

14. To compute perimeters and circumferences

   - determine which method of calculating the distance around a figure is required to solve a problem
   - compare perimeters and circumferences of various polygons and circles

UNIT V - MONEY MANAGEMENT

1. To learn a method of subtracting dollars and cents from dollar amounts
   - express a monetary value written in cents as a decimal.
   - write dollar amounts as dollars and no cents.
   - compute the change when items are paid for by $1, $5, $10, or $20 bills
2. To learn to use unit pricing in a supermarket
   - state the relationships among liquid measures: ounces; pints: quarts: gallons.
   - read and compare samples of supermarket unit price shelf markers to find the best buy.
3. To compute the actual price from the unit price
   - state the relationship between pounds and ounces in weight.
   - find the product of two decimal numbers, and round off to the nearest hundredth.
   - round off a monetary amount to the nearest cent.
   - apply the rule: actual price = unit price times weight (or quantity).
4. To compute the unit price from the actual price
   - find the quotient of a decimal number (monetary value) by a whole number expressed to the nearest cent.
   - find the quotient of one decimal number by another to the nearest hundredth.
   - apply the rule: unit price = actual price divided by the weight (or quantity).
5. To compute the weight of an item, given the actual price and unit price
   - apply the rule: weight (or quantity) = actual price divided by unit price.
6. To review unit pricing
   - calculate a missing unit price, weight or quantity, or actual cost, given the other two.
7. To compute salary based on an hourly wage
   - define "gross wage."
   - complete a payroll table which does not include any overtime pay.
8. To read a time card
   - determine the number of hours between two given clock times, including A.M. and P.M. times.

9. To compute salary based on hourly and overtime wages

   - find $1\frac{1}{2}$ times any dollar amount to the nearest cent.

   - determine a worker's overtime hours.

   - complete a payroll table which includes time and a half overtime pay.

10. To compute salary based on piecework or a piece-rate plan

   - name several jobs which pay by the "piece-rate plan."

   - complete a payroll table where the workers are paid by the piece.

11. To find the cost of an item purchased on the installment plan

   - explain why people purchase items on the installment plan.

   - compute the total cost of an item purchased on the installment plan.

   - compute the finance charge on an installment purchase.

12. To compute payments based on a step function

   - provide real-life examples of "step function" price scales.

   - determine the number of units at each price.

   - compute the total price by adding the initial charge to the product of the unit rate and the number of units at that price.

UNIT VI - PROBABILITY AND STATISTICS

1. To introduce statistics as the science of collecting, organizing and interpreting data

   - provide examples in which analysis of data is needed to make decisions in many fields: politics, retailing, science.

   - define "statistics" as the science of collecting, organizing, and interpreting data.

2. To organize and interpret statistical data using a frequency table

   - read a set of random data, and set up a system of classes (categories) for the purpose of tallying.

   - tally the data according to the scheme determined above.

   - define "frequency."

   - organize the data by completing a frequency table.

3. To define and use the concept of mode

   - define mode

   - identify the mode from frequency tables, including the class survey data table.

   - define bimodality.

4. To define and use the concept of mean

   - define mean as the sum of the values divided by the number of values (average).

   - apply the definition to find the mean, given a set of data.

5. To define and use the concept of median

   - arrange a set of data in numerical order.

   - define median.

   - apply the definition to find the median in given sets of data.

6. To introduce probability as the science of chance

   - cite uses of "chance" or probability.

   - guess (or predict) the outcomes of experiments with a spinner, dice, or coins.

   - perform an experiment, tally the outcomes, and compare the outcomes to the predictions.

7. To define probability of success and failure as ratios

   - express a probability as the ratio of two quantities:

$$\frac{\text{number of successful outcomes}}{\text{total number of possible outcomes}}$$

   - state the relationship between the number of outcomes, the number of successes, and the number of failures.

   - use the rule that the probability of success plus the probability of failure equals 1. (Calculate the probability of an event given the probability of its complement.)

   - reduce proper fractions to lowest terms.

8. To reduce fractions to lowest terms

   - write a composite number as a product of two or more factors

   - categorize numbers as prime or composite

   - identify the GCF (greatest common factor) of a pair of numbers

   - reduce a fraction to its lowest terms by dividing the numerator and denominator by their GCF.

9. To introduce the concept of a sample space

   - list all possible outcomes of a probability experiment (the sample space)

   - calculate the probability of a given outcome in an experiment after listing the sample space.

10. To use sample spaces to predict the outcomes of experiments

    - describe a sample space and determine the probability of an event before performing an experiment.

11. To determine the expected number of outcomes using the probability ratio

- calculate the predicted number of outcomes by using the rule.

  Predicted number of successful outcomes = probability of success times the number of trials

Outline of Fundamentals of Mathematics Volume 2

The following is a summary of the aims and performance objectives for each of the lessons in Volume 2 of the FM curriculum guide.

UNIT VII - Solving Simple Equations

1. To solve open sentences by trial and error
   - distinguish between a mathematical phrase and a mathematical sentence.
   - state whether or not a sentence is an open sentence.
   - solve an open sentence using trial and error.

2. To solve equations and inequalities given replacement sets
   - categorize a sentence as an equation or an inequality.
   - state whether a replacement for a variable makes an open sentence true or false.
   - use a given replacement set to find the solution set of an equation or inequality by trial and error.

3. To solve equations using addition and subtraction
   - use the appropriate inverse operation and the addition and subtraction properties of equality to find the solution to equations of the form $x + a = b$ and $x - a = b$ where a and b are integers.
   - check the solution of an equation.
   - state that addition and subtraction are inverse operations.

4. To reinforce solution of equations using addition and subtraction properties of equality
   - use the appropriate property of equality to find solutions to equations of the form $x \pm a = b$, where a and b are fractions and decimals.

5. To solve equations of the form $ax = b$ and $\frac{x}{a} = b$ where a and b are positive
   - state that multiplication and division are inverse operations.
   - demonstrate the multiplication and division properties of equality in numerical sentences.
   - use the appropriate inverse operation to solve equations of the form $ax = b$ and $\frac{x}{a} = b$ where a and b are positive.

6. To multiply and divide signed numbers
   - state and apply the properties of multiplication and division of signed numbers to obtain the appropriate products and quotients.

UNIT XII - Solid Geometry

1. To introduce solid geometric figures and the concept of volume
   - identify plane and solid objects as rectangular, square, circular, and triangular.
   - distinguish between drawings/objects which are plane or solid figures.
   - state the definition of a prism.
   - state that a prism is named by the shape of its base.
   - state that the space displaced by a solid object is different from length or area, and requires a cubic unit of measure.
   - use cubes (sugar, plastic, etc.) to measure volume.

2. To find volumes of rectangular prisms
   - use the formula $V = lwh$ to calculate the volume of a rectangular prism, where l, w, and h are combinations of whole numbers, decimals and fractions.
   - find a dimension of a rectangular prism if its volume and 2 dimensions are given.
   - apply the formula $V = lwh$ to the solution of verbal problems.

3. To find the volumes of prisms and cylinders
   - state that l x w is the area of the base of a rectangular prism, so that its volume is the product of the area of its base and its height.
   - calculate the volume of any prism using the formula V = area of base x height.
   - distinguish between prisms and cylinders.
   - find the volume of a right-circular cylinder.

4. To find the volumes of pyramids and cones
   - distinguish between pyramids and prisms by recognizing that pyramids have triangular sides and prisms have sides that are parallelograms.
   - name a pyramid by the shape of its base.
   - use the formula $V = \frac{1}{3} lwh$, or $V = \frac{1}{3}$ area of base x height to calculate the volume of a pyramid.
   - distinguish between cones (circular base) and pyramids (polygonal base).
   - use the formula $V = \frac{1}{3}$ area of base x height to calculate the volume of a cone.
   - identify a given geometric solid as a prism, a cylinder, a pyramid, or a cone.

5. To find volumes of spheres
   - distinguish the sphere from other solid figures.
   - use the formula $V = \frac{4}{3} \pi r^3$ to find volumes of spheres.

7. To solve equations of the form $\pm ax = \pm b$ and $\pm \frac{x}{a} = \pm b$
   - apply the properties of multiplication and division of signed numbers.
   - apply the techniques of equation solving to solve equations of the form $\pm ax = \pm b$ and $\pm \frac{x}{a} = \pm b$.

8. To reinforce the solution of simple equations
   - select which property of equality to use to solve a one-step equation.
   - solve any linear equation where only one property of equality must be used.

9. To solve equations of the form $ax \pm b = c$ by trial and error
   - use substitution to determine whether or not a specific value of a variable is a solution to an equation of the form $ax \pm b = c$ or $\frac{x}{a} \pm = c$.

10. To solve equations of the form $ax \pm b = c$ using properties of equality
    - state that the addition or subtraction properties must be used before the multiplication or division properties in the solution of equations of the form $ax \pm b = c$.
    - solve and check equations of the form $ax \pm b = c$.

11. To solve equations of the form $\frac{ax}{b} = c$ using properties of equality
    - write the reciprocal of any number.
    - state that dividing by any number is equivalent to multiplying by its reciprocal.
    - solve and check equations of the form $\frac{ax}{b} = c$ using properties of equality.

12. To solve equations of the form $\frac{ax}{b} \pm = d$
    - state the sequence of steps required for the solution of equations of the form $\frac{ax}{b} \pm c = d$ using inverse operations.
    - solve the equations and check the solutions.

13. To review the solution of linear equations using one or more properties of equality
    - use formulas to solve problems by substituting given values and solving the resulting equations.
    - determine if the solution satisfies the conditions of the problem.

UNIT VIII - Tables, Graphs, and Coordinate Geometry

1. To read and interpret data found in tables
   - provide examples in which presentation of data in tables is useful.
   - read common tables of data and answer specific questions relating to these data.
   - describe ways of presenting data: lists, tables, maps, graphs.
2. To read and interpret pictographs (picture graphs)
   - read the scale on a pictograph.
   - translate the symbols on a pictograph into numerical quantities.
   - extrapolate new information or form opinions using the data on a pictograph.
3. To read and interpret bar graphs
   - identify bar graphs as a type of graph that can illustrate data more accurately than pictographs.
   - translate the scale units used on a bar graph into numerical quantities.
   - read and interpret bar graphs.
   - read and interpret bar graphs with double sets of data.
4. To read and interpret line graphs
   - identify line graphs as a type of graph that illustrates trends more effectively than bar graphs.
   - read and interpret line graphs.
   - state that line graphs tend to show a continuous series, process, or tendency (optional).
5. To read and interpret circle graphs
   - identify circle graphs as a good graph to use when illustrating the subdivisions of a whole quantity.
   - read and interpret circle graphs where the subdivisions of a whole are given as numerical quantities.
   - to compare relative sizes of sectors of a circle by inspection.
6. To define percent and find percent of a number
   - express percent as the ratio of a number to 100.
   - find the percent of a number by expressing the percent as a fraction and multiplying.
   - state that the sum of all the parts of a quantity is 100%.

7. To express percents as decimals and to find a percent of a number
   - express whole number percents as decimals.
   - find a percent of a number when the percent is expressed as a decimal.

8. To locate points on a number line, to find the distance between two points and to locate a point midway between two other points
   - locate any signed number on the number line.
   - state that the number associated with a point on the number line is called its coordinate.
   - locate a point midway between two other points and determine its coordinate.
   - find the distance between two given points on the number line.
   - state that the midpoint of a line segment is the average of the coordinates of the end points.

9. To name points in the plane, using ordered number pairs
   - describe a location in a coordinate plane using two numbers (coordinates).
   - distinguish between location (3, 2) and location (2, 3), and define an ordered number pair.
   - identify coordinates of a point to the left of 0 on the horizontal number line as negative, and similarly, coordinates below 0 on the vertical number line as negative.
   - label the horizontal number line with X, and call it the x-axis (horizontal axis).
   - label the vertical number line with Y, and call it the y-axis (vertical axis).
   - locate the origin and state its coordinates.
   - read and locate the coordinates of given points on the coordinate plane, including points on the axes.

10. To locate points in the coordinate plane
   - plot points on the coordinate plane, given their coordinates.
   - state that the first number in an ordered number pair is called the x-coordinate and the second is called the y-coordinate.

11. To find the distance between 2 points in the coordinate plane
   - fine the length of a line segment that is parallel to one of the coordinate axes.
   - apply the Pythagorean Formula to find the length of a line segment that is not parallel to either coordinate axis.
   - find the distance between 2 points in the coordinate plane.

UNIT IX - Banking and Taxes

1. To compute simple interest
   - use the words deposit, withdrawal, principal, interest, interest rate, and balance to describe banking procedures.
   - explain why a bank pays interest.
   - state that interest rates are always expressed as an annual rate.
   - identify the principal, rate, and time period in a bank interest problem.
   - compute interest using the simple interest formula I = Prt, where t is an integral number of years.

2. To compute simple interest
   - express non-integral percents as decimals.
   - use the simple interest formula to calculate interest where the rate is a non-integral percent.

3. To compute simple interest involving fractional parts of the year
   - express any number of months as a fraction of a year.
   - express a non-integral percent as a fraction in lowest terms.
   - use the simple interest formula to calculate interest where the rate is a non-integral percent and the time is a non-integral number of years.

4. To understand the meaning of compound interest
   - differentiate between simple and compound interest by stating:
     (a) that when calculating simple interest:
         1. the interest is calculated all at once for the entire period
         2. the principal does not change during the period.
     (b) that when calculating compound interest:
         1. the interest is calculated for each unit of time (e.g., per year for annual compounding)
         2. the principal increases after each unit of time (e.g., this year's principal equals last year's principal plus last year's interest).
   - calculate interest compounded annually.

5. To develop the concept of equal repayments for a loan
   - calculate the interest amount charged on a loan.
   - calculate the total amount to be repaid (interest plus principal).
   - state that each installment is the quotient of the total amount borrowed divided by the number of equal installments.
   - compute each installment on a loan.

6. To introduce the idea of "bank discount"
   - compute the interest amount charged on a loan.
   - compute the actual amount of money available to the borrower (net proceeds).
   - determine whether a given amount yields sufficient net proceeds.

7. To compute the selling price when discounting
   - compute the amount of a discount if the discount rate is a unit fraction.
   - compute the selling price (net price) if the discount rate is a unit fraction.

8. To find the selling price when the rate of discount is expressed as a percent
   - convert several common percents to their fractional equivalents (10%, 20%, 25%, 33 1/3%, etc.)
   - compute the net price where the discount rate is expressed as a percent by expressing the rate as a decimal or common fraction.

9. To find the total cost of a purchase
   - calculate the sales tax (tip) by finding the product of the expenditure and the rate (of sales tax or tip).
   - express monetary values to the nearest cent.
   - compute the total cost of a purchase including tax, tip, or both.

10. To calculate the total cost of a purchase involving both discount and sales tax
   - state that sales tax is added after the sale price is calculated.
   - calculate the purchase price for goods advertised in a newspaper sale, including discounting and sales tax.

11. To solve business and consumer problems involving rates
   - state the meaning of common business terms involving the rate expressed as a percent (commission, mark-up, successive discount).
   - compute commissions, mark-ups, and successive discounts when the rates are given.

12. To determine the coordinates of the midpoint of a line segment

- state that that x-coordinate of the midpoint of a line segment is the average of the x-coordinates of the endpoints.
- state that y-coordinate of the midpoint of a line segment is the average of the y-coordinates of the endpoints.
- determine the coordinates of the midpoint of a line segment.

UNIT X - Areas

1. To find the areas of rectangles
   - provide practical examples where area is used (e.g., floor tiles, carpeting, wallpaper).
   - state that a surface (area) must be measured with a square unit, and is different from perimeter which is measured with linear units.
   - find areas of rectangles by counting the number of squares they enclose on a grid.

2. To calculate the areas of rectangles and squares
   - state that the area of a rectangle can be found using the formula $A = lw$ (or $A = bh$).
   - state that the area of a square can be found using the formula $A = S^2$.
   - use the formulas $A = lw$ and $A = S^2$
     (a) to calculate areas and,
     (b) to find a missing dimension when the area and one dimension are given.

3. To solve verbal problems involving area and perimeter
   - distinguish between area and perimeter in verbal problems.
   - solve verbal problems involving area and perimeter of rectangles and squares.

4. To solve problems involving areas of complex figures
   - calculate areas of complex regions involving rectangles by dividing the figure into component rectangles and adding.
   - solve problems involving figures of this type.

5. To find the area of a triangle
   - apply the formula $A = \frac{1}{2}\ bh$ to calculate the area of right triangles and express the area in square units.

     dentify the base and height of any triangle, where the bottom side is the base.
   - use the formula $A = \frac{1}{2}bh$ to calculate the area of any triangle.
   - use the formula $A = \frac{1}{2}bh$ to calculate the base or height of a triangle if the other quantities are given.

6. To find the areas of complex regions involving rectangles and triangles
   - calculate the areas of complex regions involving rectangles and triangles.

UNIT XI - Indirect Measurement and Scaling

1. To distinguish between direct and indirect measurement
   - describe situations in which a measurement of length or distance cannot be made directly.
   - apply formulas to find measurements of quantities indirectly by using measurements that have been made directly.

2. To understand the meaning and use of scale drawing
   - state a need for standard units of measure.
   - use a scale to determine the actual number of units represented in a drawing.
   - measure the length of line segments using a ruler.
   - determine an actual distance given a unit scale and a scaled length.

3. To identify similar figures
   - state that the enlargement or the reduction ratio is called a scale.
   - compare any two quantities with a ratio.
   - state that enlargements or reductions create figures similar to the original.
   - state that when figures are similar, the corresponding parts (lines) are in proportion.
   - identify a true proportion by:
     (a) reducing fractions and comparing them.
     (b) using the cross-product property of proportions.

4. To use proportions to find unknown sides of similar triangles
   - use the cross-product property to solve proportions (e.g., $\frac{a}{b} = \frac{x}{c}$)
   - write and solve proportions to find an unknown side in similar triangles.

5. To use proportions to solve problems involving ratios and rates
   - represent verbal problems as proportions involving rates or ratios.
   - solve proportions using the cross-product property.

6. To use proportions to solve problems involving scaling
   - represent and solve problems involving scaling by using proportions.

7. To find the area of a circle
   - state the formula used to find the area of a circle: $A = \pi r^2$.
   - apply the formula to find the areas of circles using $\pi = \frac{22}{7}$

8. To apply the formula for the area of a circle
   - apply the formula $A = \pi r^2$ to find the areas of circles using $\pi = 3.14$.

TOPICS ON RCT

1) Addition of whole numbers
2) United States currency
3) Subtraction of whole numbers
4) Multiplication of whole numbers
5) Division of whole numbers
6) Numbers written in word form
7) Rounding (whole numbers)
8) Place Value
9) Motion (R x T = D)
10) Down payments/Installment Buying
11) Mean, median, mode
12) Approximation
13) Greatest common factor and divisibility
14) Reducing fractions
15) Improper fractions ---- mixed numbers
16) Meaning of fractions
17) Fractions ---- decimals
18 Multiplication of fractions; wages; recipes
19) Division of fractions
20) Least common denominator
21) Addition of fractions
22) Subtraction of fractions
23) The ruler
24) Ratio
25) Scale drawing
26) Proportion and equivalent fractions
27) Probability
28) Decimals ---- percents
29) Rounding decimals
30) Addition of decimals
31) Subtraction of decimals; making change; deductions
32) Checking and savings accounts
33) Multiplication of decimals
34) Division of decimals
35) Meaning of percent
36) Decimals, fractions ---- percents
37) Finding a percent
38) Finding a percentage of a number
39) Taxes
40) Tips
41) Discounts and sales price
42) Interest
43) % increase
44) Commission
45) Bar graphs
46) Circle graphs
47) Line graphs
48) Pictographs
49) Angles, protractor, parallel and perpendicular lines
50) Sum of the angles of a triangle
51) Perimeters: rectangles, parallelograms, squares, triangles
52) Areas: rectangles, squares
53) Parts of a circle
54) Circumference of a circle

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55) Area of a circle
56) Prime numbers
57) Solid figures
58) Volume
59) Meaning of signed numbers
60) Number line graphs
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62) Addition of signed numbers
63) Multiplication of signed numbers
64) Division of signed numbers
65) Simple algebraic equations
66) Evaluation
67) Exponents
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LESSON PLANNING IN MATHEMATICS

1. The math teachers should prepare a detailed lesson plan for each lesson.

2. The parts of a lesson plan are: (a) Aim (b) Performance Objectives) (c) Do Now (d) Motivation (e) Development (f) Applications (g) Summary

3. The aim should be stated clearly and simply and written in question form. It should be elicited from the pupils. It need not, however, be elicited at the very beginning of the period.

4. The teacher must recognize what previous learning pupils will need as a foundation on which to base the principle to be taught in the new lesson. The do-now questions should include pre-requisite skills, defintions or procedures needed for the new lesson.

5. The motivation should be brief and to the point. It should lead to a statement of the aim of the lesson and a desire to achieve it. The motivating problem should be simple enough to be solved by the class after the development of the lesson is completed.

6. During the development of the lesson, the teacher should:
   a) maintain control of its pace and direction
   b) avoid dominating it, but promote widespread pupil participation and inter-pupil relations, as much as possible eliciting rather than telling correct answers.
   c) make use of proper questioning techniques, asking questions which are clear and unambiguous, including a number which will stimulate thought and evoke sustained responses by pupils, as well as others which will help pupils recognize shortcomings and errors in some of their answers.
   d) think through the statements that will be made, the questions to be asked and the answers expected from the pupils as the lesson develops.
   e) try to anticipate incorrect answers or unexpected questions to see where they may lead, so that the lesson may not be misdirected. An unexpected wrong answer or a surprising question can easily upset a lesson for the entire class.
   f) include the wording of statements and of questions in the lesson plan.
   g) elicit from pupils complete and accurate statements, suitably and clearly expressed, employing technical vocabulary correctly and intelligently.
   h) check that pupils understand the principle used to obtain correct answers submitted using the lesson.
   i) see that all relevant material is not neglected, and that irrelevant matters do not intrude.
   j) present a sufficient number and variety of situations before calling on pupils to formulate a generalization.

7. The teacher should prepare in advance a statement of the generalization to be reached by pupils in the lesson and of the question he/she will ask in order to elicit this generalization from the class. The statement should be in language that is clear and easily understood by pupils, and mathematically correct. Any technical vocabulary employed should be adequately reviewed. By skillful questioning, the students themselves should be helped by the teacher to formulate the generalization.

8. One of the first applications of the principle learned during the lesson should be to solve the motivating problem previously mentioned. Other applications of this principle should be presented to the class.
When pupils submit correct solutions, the teacher should see that in each the

principle used to obtain them is emphasized. The teacher should prepare in advance a large number--more than he thinks he may need--of carefully graded examples which require the use of this new principle.

9. A summary of what was accomplished during the period should be elicited from the students. The summary is a check upon whether the aim of the lesson has been achieved.

10. The presentation of the new lesson should not be at the expense of reviewing the homework. The teacher must see that there is time for both.

11. It is also preferable that a teacher plan to cover more than can be covered in a period rather than find a void in the lesson for even a few minutes.

SIGNIFICANT ASPECTS OF A MATHEMATICS LESSON

1. "Do Now" (Warm-Up) or (Seatwork)
   a. Should be related to the new lesson, not just an arbitrary exercise i.e. should include pre-requisite skills, definitions, or procedures needed for the new lesson of the day.
   b. Should be written on the board at the beginning of the lesson.
   c. Should serve as a transition between previous learnings and new.
   d. Should take no longer than five minutes to review.
   e. Should be short and attainable by students within the first few minutes of the period.
   f. Can be used to provide writing experience where appropriate by asking students to write definitions, procedures, explanations, etc.
   g. Does not have to be used in a math lesson where the lesson begins immediately.

2. Motivation
   a. Should create a <u>need to know</u> ("in order to solve this problem we must learn this new technique...") or a <u>desire to know</u> ("Why does this card trick work?")
   b. May lead from a problem or apparent paradox
   c. Is short
   d. Yields the aim
   e. Is referred to in the summary

3. Aim
   a. Should be in the form of a question.
   b. Can be easily measured for achievability.
   c. Should be written on the board and left for the entire period.

4. Homework
   a. Should be cyclical (i.e., examples from a month ago, week ago, day ago)
   b. Should be reasonable in length
   c. Should take at most 10-12 minutes to review, preferably less.
      1. Student should "carry" review.
      2. Solutions and <u>questions</u> should be written out on students' work.
      3. Board responsibilities should be assigned in advance.
      4. Can employ visual devices (overhead, colored chalk, etc.) to save time and promote clarity.
   d. Should be numbered.
   e. Should be assigned at the start of the period and written on the same area of the board or be included in a homework assignment sheet.
   f. Should include RCT math or regents review questions.
   g. Should be solved by teachers in their lesson plans.
   h. Should be checked or collected.
   i Can be referred to appropriately in the new lesson.
   j. Provides pre-requisite skills for the new lesson.
   k. May be reviewed at the beginning or the end of the period.
   l. Should include a reading assignment.

5. Development
   a. Concepts should be taught along with techniques i.e., the "why" along with the "how"
   b. One new thought should be developed at a time in the sequence of logical analysis.
   c. The problems used as models should be well chosen and graded in diffculty. i.e., they yield the desired concepts and skills to be studied.
   d. The joy and beauty of the mathematics should be revealed.
   e. Concrete models, diagrams and other visual devices should be employed.

f. The board work should be detailed, neat and written from left to right on the board.
g. Students should share a mutual discovery with the teacher.

6. Questioning
   a. Skillful questioning serves to direct activity:
      - to provoke discussion which will extend knowledge and awareness
      - to stimulate thinking
      - to measure accomplishment by testing information and understanding
      - to seek evidence of reflective thinking and judgments based on proof.
   b. Skillful questioning should keep the student more alert, more critical, more active. Therefore, frame questions of graded difficulty in order to maintain interest, and appeal to the varying abilities of students. Distribute questions, calling on non-volunteers as well as volunteers. Handle the erroneous answers of the weaker students without belittling their attempts. Encourage topical, sustained answers.
   c. To stimulate critical thinking, questions should evoke sustained answers. Therefore, it is poor practice to ask too many questions or to use only factual questions.
   d. After asking a difficult question, it is wise to pause before calling on a student. Such a pause will provide time for thought and discourage the quick, glib answer. The pause also provides time for students who think more slowly to react.
   e. Once a student begins a developed answer, permit him to finish without interrupting him.
   f. Since at all times responses should be directed to the class rather than to the teacher, other class members are free to challenge the accuracy of these responses. (Such automatic procedures result, of course, from constant, patient training in the early weeks of the term and from reminders given constantly throughout the term.) Where an individual--teacher--individual recitation is used, general student participation is impossible, and lack of respect for others as well as inattention (day-dreaming, talking, etc.) will result. The teacher has to keep the group in the picture even when dealing with an individual problem.

   g. Criteria for judging questions asked in the classroom

      1. They should be simple and direct in language.
      2. They should be specific in directing attention to the particular problem under consideration.
      3. They should follow a planned order which develops the train of thought in a logical sequence.
      4. They should place the burden of thinking upon the student (and, therefore, should be relatively few in number).
      5. They should be so framed that the answer may not be a simple 'yes' or 'no'.
      6. There should be included in the lesson questions which require students to draw conclusions.
      7. Questions should be adapted to the varying levels of ability in the class.
      8. They should be sufficiently difficult to arouse effort, yet not so difficult as to preclude a response.
      9. There should be a proper balance between fact and thought questions.
      10. Questions should not be repeated. (If you must have one repeated, have a student repeat it).

7. Applications and Drill Problems
   a. Should be graded from simple problems to complex
   b. Should be reviewed on the board by students in the class
   c. Should provide enrichment for talented students
   d. Should be pre-planned (and solved completely in teacher's lesson plan).

8. Summary
   a. Should be elicited from students
   b. Often yields a "recipe" type of problem-solving procedure. Should include key concepts as well.
   c. Can be generated from a "tour" of board work from left to right.
   d. Should refer to the aim.
   e. A medial summary, elicited from the class, should be provided prior to application and/or drill.
   f. A final summary elicited from the class, should be provided after the application problems.

9. Miscellaneous
   a. Teacher should circulate around the room monitoring notebooks and assisting students with individual problems.
   b. Teachers' board work should serve as a model. They should use a ruler, be neat and write out full sentences. Diagrams should be easy to read.
   c. Teachers should have worked out solutions to all homework problems and problems used during the period.
   d. All problems used should not be test oriented.
   e. Timing of a typical lesson:
      5 minutes--"Do Now"
      5-10 minutes--Homework review
      12-20 minutes--Development
      7 minutes--Drill and/or applications.
      2 minutes--Summary

TECHNIQUES OF MOTIVATION

1. Indicate a void in student's knowledge.
2. Show a sequential development.
3. Present a challenge.
4. Indicate the usefullness of a topic.
5. Use recreational mathematics.
6. Tell a pertinent story.
7. Get students actively involved in justifying mathematical curiosities.
8. Use teacher-made or commercially prepared materials.

Posamentier, Alfred & Stepelman, Jay, Teaching Secondary School Mathematics (Charles Merrill Publishing Company, 1981).

THE USE OF THE "DO NOW" IN A MATHEMATICS LESSON

I. Why do we give "do now" problems?

A. To review pre-requisite skills, concepts, vocabulary or procedures that will be needed for the new lesson.
B. To provide opportunities for integrating writing in the mathematics class.
C. To set a routine of students beginning their work as soon as they enter the room. This also sets a tone that the entire period is spent on instruction and not irrelevant activities.
D. To provide the teacher with an opportunity to handle clerical matters and yet not interfere with student learning. This is only one of several opportunities that the teacher has to handle such things as taking attendance without having students unoccupied in a learning activity.

II. What are some of the characteristics of a well chosen "do now?"

A. It reviews pre-requisites skills, concepts, vocabulary or procedures needed in the new lesson.
B. It is within the ability of the class to answer.
C. It can be copied and completed by the class in no more than the first five minutes of the period.
D. It can be used as an opportunity to provide mathematics students with writing experiences.

III. What should we avoid when we use a "do now?"

A. Don't allow it to take too much time.
B. Don't allow it to become a review lesson. Review lessons, like all lessons, must be motivated and developed. The "do now" should NOT become the lesson.
C. Don't include the new topic in the "do now" assuming that the students will know how to do it. If the students can answer questions on the new topic, perhaps, you do not need to teach that lesson.
D. The "do now" is not the motivation, although it may contain part of your motivation.
E. The "do now" is not independent from the rest of the lesson. There should be a <u>transition</u> from the "do now" to what follows. It should be referred to at some point in the lesson. If it is not referred to, perhaps it was not needed.
F. Don't use a "do now" if the lesson does not call for it. A lesson can start immediately without a "do now".
G. Don't be redundant. If your homework assignment has reviewed your pre-requisite skills, you need not do it again in a "do now".

SUGGEST AN APPROPRIATE "DO NOW"

From FM1:

1. Aim: How can we compare two rational numbers with unlike denominators?

2. Aim: What is meant by the Pythagorean relationship?

From FM2:

1. Aim: How can we solve equations of the form $ax=b$ and $\frac{x}{a} = b$ where a and b are positive?

2. How can we solve problems involving areas of complex figures?

From MQ1:

1. Aim: How can we find the product of a polynomial by a monomial?

From MQ2:

1. Aim: How do we find the product of two binomials?

2.. Aim: How do we factor a quadratic trinomial?

From MQ3:

1. Aim: How do we do simple proofs involving bisectors and midpoints?

2. Aim: How can we prove the converse of the isosceles triangle theorem?

3. Aim: How do we prove that products of the lengths of line segments are equal?

4. Aim: What are the special relationships among the sides of a 30-60-90 triangle?

From MQ4

1. Aim: How can we solve quadratic equations by factoring?

2. Aim: How do we graph a quadratic relation on a set of coordinate axes?

3. Aim: How do we use the "completing the square" technique to solve quadratic equations?

4. Aim: What is the relationship between the coefficients of a quadratic equation and the sum and product of its roots?

FROM LESSON PLANNING TO PERFORMANCE

A. A lesson may not be successful despite the fact that the plan included a worthwhile aim, a challenging motivation and thought-provoking questions. The following are suggestions to help you insure success in the execution of your daily plans:

1. You must be prepared to answer questions that may be raised by the pupils. The answering of such questions may require a knowledge of your subject that goes beyond the content of the textbook. A potent motivation for learning on the part of the pupils can be the scholarship of the teacher.

2. You must be prepared to develop the lesson along a path different from the one you set down in your plan. This way come about because of the answers that are presented in response to your motivation. The class may suggest a better aim than the one you have prepared.

3. You must make provision for the active participation of the greatest number of pupils in each lesson. Do not permit a few students to dominate or by watching demonstrations. To insure the greatest amount of participation it is suggested that:

   a. You establish the rule that hands be raised only when you request volunteered answers or when a pupil wishes to ask a question.

   b. You call on the slow, passive and shy pupils to repeat answers, to answer simple questions, and to evaluate answers.

4. You should make the entire period, not only the motivation time, a stimulating and challenging experience for the class. Suggested procedures to realize this goal are:

   a. Sustain the motivation throughout the period.

   b. Employ questions that stimulate thinking and that produce interaction among the members of the class.

   c. Use demonstrations and experiments that excite the interest of the pupils.

5. Make certain that, throughout the period, understanding and retention are achieved on the part of the pupils. You should recognize that answers may be inaudible, fragmentary, or poorly-phrased; that members of the class may be inattentive at times; and that all students may not comprehend answers or board demonstrations. The following suggestions are given to insure understanding and retention:

   a. Insist that pupils speak in an audible tone to the majority of the class. If you feel that an answer has been inaudible to some members of the class, have it repeated by a student.

   b. If an answer lacks details, if it does not cover all facets of the problems, or if it is totally or partially incorrect: call on other pupils to supplement or to evaluate the answer. If necessary, pose another question that aims at soliciting additional information.

   c. If, after calling on a few pupils, you are unable to elicit the desired response--rephrase the question. If the response is still not forth-

coming, present the answer yourself. Do not waste valuable teaching time in an attempt to secure an answer to any one question.

d. Prepare pupils for the observing of experiments and of board demonstrations. During the execution of experiments and demonstrations ask questions and solicit questions.

e. Call on a student to provide a summary after a number of answers have been presented in response to a particular question.

f. Include drills, medial summaries and final summaries and applications in every lesson.

PRACTICE MAKES PERFECT

A. Many roads lead to Rome. There is no one correct method to teach a particular topic or to achieve a particular aim. The plan used successfully by one teacher with a particular class may not achieve the same results when used by another teacher. You will, as a result of your own planning, discover the most effective methods to be employed in teaching each topic with different classes.

B. More effective teaching on your part will result in better pupil achievement, better class discipline and an easier and a more satisfying teaching day for you. To improve on your lesson plan it is suggested that:

1. You critically evaluate each lesson in terms of the interest of the students and of the learnings that took place. If your plan has not proved effective during the period that you taught it, try to revise it before you teach the same lesson to another class.

2. Secure suggestions from veteran teachers in your department for the teaching of particular lessons.

3. You pre-plan lessons with your supervisor.

4. You observe more experienced teachers.

5. You re-write your lesson plans each year. Classes change; the world changes. An aim or a motivation that is timely in one year may have no appeal a different year.

6. You consult frequently with your subject supervisor.

Practice will not make you a perfect planner or a perfect teacher. There is no such person. What it will do is make you an interested teacher, an inquiring teacher and therefore a better teacher. The secret of your success will lie in your own efforts toward self-improvement.

MAXIMIZING INSTRUCTIONAL TIME IN THE CLASSROOM

Listed below are a number of suggestions for making the most of each and every teaching and learning experience.

Before the lesson begins:

1. Plan carefully and fully in advance of each lesson.

2. Have the homework assignment on the board as the class comes into the room.

3. Use homework assignment sheets, if at all possible, to save time.

4. Organize boards in advance to include the lesson topic and a "Do-Now" if appropriate for the lesson. Section the boards in advance if appropriate to the lesson.

5. Arrange for books and materials to be ready for distribution so that time is not wasted, nor is the rhythm of the lesson disrupted once the lesson is under way.

6. Use students as monitors to distribute materials and to write at the board whenever possible.

7. Collect homework after it has been gone over by the class. To go through each and every row can take away valuable instructional time.

Once the lesson is under way:

1. Limit the motivation to 3-4 minutes and elicit the aim within that time frame as well.

2. Take attendance through a seating plan while the class is engaged in reading and writing activities.

3. Limit copying of materials at the board to simple outlines and notes.

4. Limit the time spent on "Do-Now" to no more than 5 minutes.

   Avoid digressions which are not germane to the lesson or do not serve to enrich or enhance the lesson's purpose or pupils' understanding or appreciation.

6. Do not see individual pupils during the class when their problem is an individual one, inappropriate to class concerns.

7. Use able students to tutor those students who have fallen behind.

At the end of the lesson:

1. Make sure that you summarize the lesson within the last 3-5 minutes so that what students should have derived from the lesson is clear.

2. In the lesson plan provide for "extra-credit" work or enrichment activities to allow for lessons running short.

3. Have latecomers see you at the end of the period. Do not interrupt the lesson to deal with each latecomer as he or she enters.

4. Limit announcements and routine items to the last minute rather than at the beginning of the lesson.

The above list is by no means exhaustive. You may want to add to it, to refine it or modify it, but what is imperative is that teachers avoid the pitfalls that lead to so much time loss in the classroom.

Sample Lesson Plan in Fundamentals of Mathematics

(Based on Lesson 12, page 121 of Fundamentals of Mathematics, Part I.)

Aim: How do we calculate the circumference of a circle?

Performance Objectives: (Same as those in curriculum guide), plus:

- Define pi as the ratio of the circumference of a circle to its diameter.
- State that the value of pi is constant for all circles.

Vocabulary: (Same as in curriculum guide.)

Challenge Problem:

Write a sentence that explains how you would measure the distance around a circular ice skating rink.

Development:

Have students read their sentence.

Ask: What do we call the distance around a circle? Elicit: Circumference.

Demonstrate use of rolling marker to measure distances. Distribute printed sheet of circles and lengths of beaded light-pull chain. Make circles so that diameter of one is twice diameter of other. Have students measure circumferences and diameters of circles by placing chain around circle and along diameter and counting beads in the length of chain. Have one student do the same at the overhead projector on transparency of same circles.

Make a chart on board, and have students do the same on their printed sheets:

	Circle 1	Circle 2
Circumference (number of beads)		
Diameter		

Ask: How do the diameters of the two circles compare to each other? How do the circumferences of the two circles compare to each other? Do you think that it will always happen that the circumferences and diameters of circle are in proportion? What is the ratio of the circumference of the first circle to its diameter? Have students divide. What is the ratio of circumference to diameter in the second circle? Have students divide. Add to summary chart on third line.

	CIRCLE 1	CIRCLE 2
Ratio circumference/diameter		

Define pi as the constant 3.14.

Ask: How can we use the value of pi to calculate the circumference of a circle whose diameter is known?

Draw circle of diameter 12 inches on board with compass.

Ask: What is the circumference of this circle?

Elicit that multiplying 12 by 3.14 will give the circumference of the circle. Verify by measuring with a piece of stiff electrical wire at the board.

Use #2 in development in curriculum guide.

Do Applications in curriculum guide.

Follow summary in curriculum guide.

Homework #27:

pp. 121-122: 2, 5a, c, 8, 11, 15

p. 78: 11, 14

p. 52: 9

Write a paragraph to explain how you could show someone the value of pi by measuring a phonograph record with a tape measure.

Analyze The Following Lesson Plan
Using "Significant Aspects of a Mathematics Lesson"

Aim: The angles of a triangle

Performance Objectives:

- Understand the sum of the angles of a triangle.
- Appreciate that the sum never changes form triangle to triangle.

Do Now: Page 113: 16, 17, 18, 19, 20, 21

Development:

What is the sum of the angles of a triangle?
Student measures a triangle at the board. Add up the three measures. If not 180, tell students that it is supposed to be 180, error due to measuring instruments.

Class exercises: Page 115: 10 to 20, all even.

Summary:

What is the sum of the angles of a triangle?
Which of the following sets of angles may be the angles of a triangle? (Make up four sets of angles, only one is possible.)

Homework:

Page 115: 11-21, all odd. 22-28.

Compare the sample lesson plan presented in this manual on "How do we calculate the circumference of a circle" with the lesson plan that deals with "The angles of a triangle". How did the writer of the "Circumference" lesson plan adapt the materials provided in the curriculum guide for Fundamentals of Mathematics Part I?

Situations for Discussion by New Teachers: Lesson Planning

1. You discover, after the first two weeks of school, that you are already two lessons behind a more experienced teacher who is teaching the same course. What do you do?

2. You find, after the first 25 minutes of a 40 minute lesson, that you have reached the end of your lesson plan. What do you do?

THE ART OF QUESTIONING

Effective questioning is the tool with which teachers develop logical thought, stimulate the imagination, bridge the gap between previous learning and new learning and give direction to the learning process. When skillfully phrased and strategically used, questions become one of the teacher's most flexible instructional tools. On the other hand, poor questions and poor questioning techniques interfere with learning by creating confusion, discouraging pupil involvement and inhibiting the sequential development of concepts.

General characteristics of effective questions:

- They are kept short.
- They are simply phrased and direct in language.
- They are logically sequenced, the response to one question leading into the next, with each question calling for only one or two points in the chain of development.
- They are designed to require pupils to think, rather than respond instantly with an association. Whenever possible, they elicit a sustained response rather than a one- or two-word answer.
- They are frequently personalized to grasp the pupils' attention: "How would you explain to a classmate who was absent yesterday our method for changing common fractions to decimal fractions?"

Twelve Types of Questions to Avoid: A Dirty Dozen

Teachers sometimes develop ineffective questioning habits that are difficult to break unless conscious examination in precisely what kinds of questions they have been asking takes place. Ineffective questions come in many varieties, and are ineffective for various reasons. We have all heard them, and have perhaps used some of them. Here is a brief compilation of some of them:

(1) **The Multiple or Overlaid Question:**

Example: What is the number above the fraction bar called, and what is the number below the fraction bar called? How do we reduce fractions?

Criticism: By asking for more than one response, the teacher is limiting participation to only those pupils who know all of them. Many more pupils might be able to respond if a single question were asked at a time.

Suggestion: Ask the three questions separately, waiting until one question is answered before going on to the next.

(2) **The Vague or Ambiguous Question:**

Example: How about rectangles?

Criticism: The question doesn't direct pupil thinking in any particular direction. A multitude of irrelevant responses can be made.

Suggestion: What kind of angles do the sides of a rectangle form?

(3) **The Choral Question:**

Example: Were we right to multiply by 100?
(Y-e-e-e-e-s!)

Criticism: Pupils are not made to feel personally responsible for the accuracy of their responses. They are encouraged to hide behind the anonymity provided by choral response. Different responses to the same question are uttered together, creating confusion for individual pupils who may not know which one is being accepted as correct.

Suggestion: Do you think we were right to multiply by 100, ... Joseph?

(4) **The Leading Question:**

Example: There are 6 fours in 24, aren't there?

Criticism: A pupil who is not sure will promptly agree with the teacher without thinking about his or her response. The pupil has been invited merely to nod his or her head in acquiescence.

Suggestion: How many fours are there is 24, ... Sherry?

(5) **The Tugging Question:**

Example: The area of a rectangle is found by multiplying the length by the, ___ ...? W___, w___. Come on now; it starts with "W".

Criticism: The question reduces the discussion to a guessing game, often not even based on the subject matter but on the recall of words by virtue of their sounds.

Suggestion: How is the area of a rectangle found? OR: What two measures do we multiply to get the area of a rectangle?

(6) **The Teacher-Oriented Question:**

Example: Who can tell me how many places to move the decimal point when multiplying by 1000?

Criticism: The question contributes to a feeling that the class is organized around the teacher, and that what is most important is satisfying the teacher's requirements. It does not contribute to a group feeling of a class working together to learn.

Suggestion: Who can tell us how many places to move the decimal poin when multiplying by 1000?

(7) **The Whip-Lash Question:**

Example: The unit of measure that is equal to 1000 meters is called the what?

Criticism: The question is not perceived as a question by the pupils since it does not begin with an interrogative. It sounds as though the teacher is making a declarative statement. Only at the last word do pupils learn that they are being asked for a response, and thus have no time in which to think about it.

Suggestion: Include one of the words: how, what, why, when, etc. near the beginning of every question. "What is the name we give to the unit of measure that is equal to 1,000 meters?"

(8) **The Yes-No Question:**

Example: Is the radius of a circle twice the diameter?

Criticism: Every pupil has a 50% chance of answering correctly by merely guessing "yes" or "no." The question does not encourage thought, nor does it encourage pupils to use the vocabulary that they have learned.

Suggestion: By what number would you multiply the measure of the radius to find the measure of the diameter?

(9) **The Echo Question:**

Example: There are four quarters in a dollar. How many quarters are there in a dollar?

Criticism: The question calls for only the most rudimentary form of response: parroting. It does not stimulate thought, and a pupil can answer it correctly without really understanding the concept.

Suggestion: Wait for some time to pass before asking pupils to recall a previously developed fact.

(10) **The Question Addressed to a Specific Pupil:**

Example: Robert, can you move the hands on this clock to make it read one-thirty?

Criticism: By starting with the name of a specific pupil, the teacher has turned the discussion into a one-to-one exchange. Other pupils are not encouraged to think about their responses, since the question appears to have been designed for only a particular pupil.

Suggestion: Ask the question, pause to give all pupils time to consider their responses, and then call on a particular pupil. Who can move the hands on this clock to make it read one-thirty? ... Robert?

(11) **The Guessing Question:**

Example: Is 25% equal to one-half or one-quarter?

Criticism: Like the Yes-No question, this type of question limits thinking and creates a situation in which any pupil, regardless of his or her understanding of a concept, has a 50-50

chance of answering correctly.

Suggestion: What common fraction is 25% equal to? OR: Is 25% equal to one-half or one-quarter, and explain why you made your choice.

(12) **The Re-phrased Question:**

Example: What name do we give a half-circle? I mean, when you draw a diameter, it divides the circle into two parts. What do we call each part? When a circle is divided into two halves, what do we call each half?

Criticism: Pupils are still dealing with the first version of the question when they are presented with second and third versions. While listening to these re-phrasings of the original question, thought is diverted and clarity is inhibited.

Suggestion: Ask one good question clearly expressed and remain silent while pupils are considering their responses to it. For example: If a diameter is drawn in a circle, it divides the circle into two equal parts. What names is given to each half-circle?

Pupil Responses

Related to the art of effective questioning is the skill with which the teacher reacts to, or accepts pupil responses. Pupils can be encouraged to formulate their responses in such a way as to make them of value to the entire class. Teachers can further encourage pupil involvement and enthusiasm in the learning process by the ways in which they use pupil responses. Some general guidelines follow:

- Encouraging pupils to comment on each other's responses involves pupils in the learning process to a greater extent than reserving the right to comment as an exclusive right for the teacher: Do you agree with Mario's explanation? How would you correct Mary's answer? What is another way of solving Renee's problem?

- Following up an incorrect response with a sequence of questions designed to help the pupil correct his own answer is much more encouraging than merely skipping over it.

QUESTIONING IN THE MATHEMATICS CLASSROOM

You will find below some questions which might be asked by teachers of Fundamentals of Mathematics. Some of them are well constructed, in good form and in good English. Others are faulty in one way or another. Criticize each question according to the following pattern.

a. If you think the question is a good one, indicate this fact, and explain why you think it is good.

b. (1) If you think the question is a poor one, indicate this fact, and explain why you think it is poor, and

(2) rephrase the question in a way which will make it satisfactory.

1. Tell me, the difference between $4\frac{1}{3}$ and $1\frac{2}{3}$.
2. The L.C.D. of $\frac{1}{2}$ and $\frac{5}{6}$ is what?
3. How about triangles?
4. Is -3 bigger or smaller than 0?
5. Ana, do you know the square root of 16?
6. What is the procedure for solving problems involving areas of complex figures?
7. What percent of the circle is paid in royalties and how can we express 20% as a equivalent decimal?
8. Explain why a triangle can not contain two obtuse angles.
9. To change a percent to a decimal you move the decimal point two places to the _____?
10. Does a right angle contain 90°?
11. What advantages are there in using a bar graph instead of a pictograph to interpret data?
12. How can we determine, if a proportion is true?

QUESTIONING IN THE MATHEMATICS CLASSROOM

You will find below some questions which might be asked by teachers of Consumer Mathematics. Some of them are well constructed, in good form and in good English. Others are faulty in one way or another. Criticize each question according to the following pattern.

a. If you think the question is a good one, indicate this fact, and explain why you think it is good.

b. (1) If you think the question is a poor one, indicate this fact, and explain why you think it is poor, and

(2) rephrase the question in a way which will make it satisfactory.

1. An example of a rectangular solid is what?
2. Explain why the cost of using a public transportation would change monthly?
3. What about buying a new car?
4. We compute the circumference of a circle, how?
5. When finding the area were we right to multiply length x width?
6. Describe the procedures that would be necessary to reconcile your checking account?
7. Is the mode the number that appears most frequently?
8. Compare the similarities between the construction of a bar graph and a line graph.
9. Who can tell me the cost of the car with the following options; air conditioning and automatic transmission.
10. In proving two triangles similar must one or two angles be congruent?
11. How can we calculate the elapsed time of a trip if we start at 10:15 AM and arrive at 1:35?
12. Susan, what is the Pythagorean Theorem?

QUESTIONING IN THE MATHEMATICS CLASSROOM

You will find below some questions which might be asked by teachers of Sequential Mathematics Course I. Some of them are well constructed, in good form and in good English. Others are faulty in one way or another. Criticize each question according to the following pattern.

a. If you think the question is a good one, indicate this fact, and explain why you think it is good.

b. (1) If you think the question is a poor one, indicate this fact, and explain why you think it is poor, and

(2) rephrase the question in a way which will make it satisfactory.

1. The base of x^2 is what?
2. The base angles of an isosceles triangle are equal in measure aren't they?
3. Is 19 a prime number?
4. What procedure should be followed when solving an equation in which like terms appear in either member of the equation?
5. Wouldn't you say that if two angles are vertical angles, they are equal?
6. Are opposite angles of a parallelogram equal or supplementary?
7. Who can tell us how to factor $4x^3 + 6x^2$?
8. How does the procedure for factoring $(a^2 - 9)$ differ from factoring $x^2 + 5x - 6$?
9. James, what is the sum of -6 and +3?
10. Who can tell me, what is sum of the measures of the angles of a triangle?
11. What additional procedures must be followed when graphing an inequality equation?
12. What about probability?

QUESTIONING—COMMON MISTAKES

1. The teacher does not distribute the questions evenly among all students.
2. The teacher cannot be heard by all the students.
3. The teacher does not use English correctly when asking questions.
4. The teacher asks irrelevant or meaningless questions.
5. The teacher repeats the answer to the questions he/she asks.
6. The teacher calls on the student first and then proceeds to give the question.
7. The teacher compounds his/her questions, that is, he/she asks two questions successively.
8. The teacher asks a question which commands a Yes or No answer.
9. The teacher asks questions that are too difficult for the students in the class.
10. The teacher does not call on volunteers.
11. The teacher calls on volunteers to the exclusion of non-volunteers.
12. The teacher does not encourage the shy students to answer.
13. The teacher encourages unison answers by using questions that require only a one-word answer.
14. The teacher does not utilize thought provoking questions.
15. The teacher does not correct the incorrect English used by a student in answering a question.
16. The teacher does not grade questions in difficulty to meet the readiness of the class.
17. The teacher does not wait for an answer and give enough time for thought.
18. The teacher does not commend students for correct answers.
19. The teacher does not hold the students responsible for listening to the questions and to the answers.
20. The teacher does not encourage the students to ask questions.
21. The teacher does not use questions t emphasize or summarize a lesson.

Situations for Discussion by New Teachers: Questioning

1. After your first formal observation, your supervisor says that, while you were doing a commendably diligent job, you were monopolizing class discussion and were simply talking too much. What do you do?

2. After you have asked the question, "What is the relationship between the angles of a triangle?" A student who rarely answers vigorously waves his hand and answers, when called upon, "They are congruent." What do you do?

IMPROVING STUDENTS' THINKING SKILLS IN MATHEMATICS CLASSES THROUGH QUESTIONING

Skillful questions serves:

- to direct activity
- to promote discussion which will extend knowledge and awareness
- to stimulate thinking
- to measure accomplishment by testing information and understanding
- to promote student interaction

Devising Thought Provoking Questions:

When interested in developing ideas, concepts, insights, and interrelationships, teachers should use questions that ask: how, why, illustrate, show, summarize, compare, contrast, explain, describe, evaluate, etc.

Questions that promote students' thinking skills:

Explain the ...

What do you think that?

How do you think that?

How can _____'s answer be corrected?

What if ...?

Can you find another way to explain that?

Can you convince me?

What reasons or evidence ...?

How are ...similar? How do they differ?

Compare ...?

Where does _____lead you?

Why did you ...?

How did you determine that the answer is correct?

What's wrong with the statement ...?

Correcting Incorrect Statements:

By appropriate questioning the teacher can correct partially or fully erroneous statements. For example, the teacher may ask the entire class:

- With which parts of John's answer would you disagree? Why?
- What facts have we learned that tend to contradict Bill's answer?
- What's wrong with the answer just given?
- How might we revise Jill's answer to make it more accurate?

Remember, the task of the teacher is to give credit for the part of the answer that is correct and attempt to have the incorrect part of the answer improved by questioning. e.g., How could you improve the answer just given?

Questions from Students:

In order to distribute the responsibility for learning and make the class share the process, the teacher should refer students' questions to the entire class by asking: Who can help Donald by responding to his question?

National Council of Teachers of Mathematics

AN AGENDA FOR ACTION

Recommendations for School Mathematics of the 1980s

An Agenda for Action

Recommendations for School Mathematics of the 1980s

The National Council of Teachers of Mathematics recommends that

1. problem solving be the focus of school mathematics in the 1980s;
2. basic skills in mathematics be defined to encompass more than computational facility;
3. mathematics programs take full advantage of the power of calculators and computers at all grade levels;
4. stringent standards of both effectiveness and efficiency be applied to the teaching of mathematics;
5. the success of mathematics programs and student learning be evaluated by a wider range of measures than conventional testing;
6. more mathematics study be required for all students and a flexible curriculum with a greater range of options be designed to accommodate the diverse needs of the student population;
7. mathematics teachers demand of themselves and their colleagues a high level of professionalism;
8. public support for mathematics instruction be raised to a level commensurate with the importance of mathematical understanding to individuals and society.

National Council of Supervisors of Mathematics

oblem Solving

Learning to solve problems is the principal reason for studying mathematics. Problem solving is the process of applying previously acquired knowledge to new and unfamiliar situations. Solving word problems in texts is one form of problem solving, but students also should be faced with non-textbook problems. Problem-solving strategies involve posing questions, analyzing situations, translating results, illustrating results, drawing diagrams, and using trial and error. In solving problems, students need to be able to apply the rules of logic necessary to arrive at valid conclusions. They must be able to determine which facts are relevant. They should be unfearful of arriving at tentative conclusions and they must be willing to subject these conclusions to scrutiny.

Applying Mathematics to Everyday Situations

The use of mathematics is interrelated with all computation activities. Students should be encouraged to take everyday situations, translate them into mathematical expressions, solve the mathematics, and interpret the results in light of the initial situation.

Alertness to the Reasonableness of Results

Due to arithmetic errors or other mistakes, results of mathematical work are sometimes wrong. Students should learn to inspect all results and to check for reasonableness in terms of the original problem. With the increase in the use of calculating devices in society, this skill is essential.

Estimation and Approximation

Students should be able to carry out rapid approximate calculations by first rounding off numbers. They should acquire some simple techniques for estimating quantity, length, distance, weight, etc. It is also necessary to decide when a particular result is precise enough for the purpose at hand.

Appropriate Computational Skills

Students should gain facility with addition, subtraction, multiplication, and division with whole numbers and decimals. Today it must be recognized that long, complicated computations will usually be done with a calculator. Knowledge of single-digit number facts is essential and mental arithmetic is a valuable skill. Moreover, there are everyday situations which demand recognition of, and simple computation with, common fractions.

Because consumers continually deal with many situations that involve percentage, the ability to recognize and use percents should be developed and maintained.

Geometry

Students should learn the geometric concepts they will need to function effectively in the 3-dimensional world. They should have knowledge of concepts such as point, line, plane, parallel, and perpendicular. They should know basic properties of simple geometric figures, particularly those properties which relate to measurement and problem-solving skills. They also must be able to recognize similarities and differences among objects.

Measurement

As a minimum skill, students should be able to measure distance, weight, time, capacity, and temperature. Measurement of angles and calculations of simple areas and volumes are also essential. Students should be able to perform measurement in both metric and customary systems using the appropriate tools.

Reading, Interpreting, and Constructing Tables, Charts, and Graphs

Students should know how to read and draw conclusions from simple tables, maps, charts, and graphs. They should be able to condense numerical information into more manageable or meaningful terms by setting up simple tables, charts, and graphs.

Using Mathematics to Predict

Students should learn how elementary notions of probability are used to determine the likelihood of future events. They should learn to identify situations where immediate past experience does not affect the likelihood of future events. They should become familiar with how mathematics is used to help make predictions such as election forecasts.

Computer Literacy

It is important for all citizens to understand what computers can and cannot do. Students should be aware of the many uses of computers in society, such as their use in teaching, learning, financial transactions, and information storage and retrieval. The "mystique" surrounding computers is disturbing and can put persons with no understanding of computers at a disadvantage. The increasing use of computers by government, industry, and business demands an awareness of computer uses and limitations.

HOW TO SOLVE IT

First.
You have to *understand* the problem.

UNDERSTANDING THE PROBLEM

- *What is the unknown? What are the data? What is the condition?*
- Is it possible to satisfy the condition? Is the condition sufficient to determine the unknown? Or is it insufficient? Or redundant? Or contradictory?
- Draw a figure. Introduce suitable notation.
- Separate the various parts of the condition. Can you write them down?

Second.
Find the connection between the data and the unknown. You may be obliged to consider auxiliary problems if an immediate connection cannot be found. You should obtain eventually a *plan* of the solution.

DEVISING A PLAN

- Have you seen it before? Or have you seen the same problem in a slightly different form?
- *Do you know a related problem?* Do you know a theorem that could be useful?
- *Look at the unknown!* And try to think of a familiar problem having the same or a similar unknown.
- *Here is a problem related to yours and solved before. Could you use it?* Could you use its result? Could you use its method? Should you introduce some auxiliary element in order to make its use possible?
- Could you restate the problem? Could you restate it still differently? Go back to definitions.
- If you cannot solve the proposed problem try to solve first some related problem. Could you imagine a more accessible related problem? A more general problem? A more special problem? An analogous problem? Could you solve a part of the problem? Keep only a part of the condition, drop the other part; how far is the unknown then determined, how can it vary? Could you derive something useful from the data? Could you think of other data appropriate to determine the unknown? Could you change the unknown or the data, or both if necessary, so that the new unknown and the new data are nearer to each other?
- Did you use all the data? Did you use the whole condition? Have you taken into account all essential notions involved in the problem?

Third.
Carry out your plan.

CARRYING OUT THE PLAN

- Carrying out your plan of the solution *check each step.* Can you see clearly that the step is correct? Can you prove that it is correct?

Fourth.
Examine the solution obtained.

LOOKING BACK

- Can you *check the result?* Can you check the argument?
- Can you derive the result differently? Can you see it at a glance?
- Can you use the result, or the method, for some other problem?

PROBLEM SOLVING STRATEGIES

The emphasis in the strategies that follow is not on how to solve problems, but on how to read and study problems so that they may be solved. Students should be given the opportunity to try several sets of strategies. Note problem solvers get stuck because they have fixed on a single procedure or approach.

The following are guidelines for teachers in helping students solve problems:

1. Make sure students understand the problem.
   a. Do the students understand the meanings of the terms in the problem?
   b. Do the students take into consideration all the relevant information?
   c. Can the students indicate what the problem is asking them to find?
   d. Can the students state the problem in their own words? If appropriate, can the students explain the problem in terms of a sketch?
2. Help students gather relevant thought material to assist in creating a plan.
   a. Assist the students in gathering information by having them analyze the given conditions (and sometimes the assumed solution).
   b. Help students obtain information by analyzing an analogous problem.
   c. When students have become discouraged by pursuing an unproductive approach, help them view the problem from a different perspective.
3. Provide students with an atmosphere conducive to solving a problem.
4. Once students have obtained a solution, encourage them to reflect on the problem and the means of solution.
   a. If possible have students verify solutions that have not been established deductively.
   b. Encourage students to seek and present alternate ways of solving a problem.
   c. Challenge students to investigate variations of the given problem.

CHECKLIST FOR CREATING A PROBLEM SOLVING ATMOSPHERE

The purpose of the following survey is to examine your classroom teaching to determine whether the learning of problem solving is taking place. If you will honestly and thoughtfully mark all of the statements that follow, you will be able to be more aware of a classroom atmosphere that includes a emphasis on problem solving.

DO YOU		YES	NO
1.	create an impression that the teacher is also investigating the problem?	____	____
2.	foster an exchange of ideas among students.	____	____
3.	ask a sequence of questions which lead students to discover a concept?	____	____
4.	encourage students to explore original ways of solving problems?	____	____
5.	make clear that a variety of ways exist to solve problems by asking, "Can we think of another way of doing this?"	____	____
6.	focus students' attention on process of thinking?	____	____
7.	encourage students to make up problems?	____	____
8.	promote divergent thinking?	____	____
9.	avoid lecturing?	____	____
10.	avoid asking, "Are there any questions?" or "Is there anyone who doesn't understand?"	____	____
11.	avoid allowing students to shout out answers spontaneously?	____	____
12.	avoid using responses of more able students to dictate pace of class?	____	____
13.	ask class to react to an answer given by a student rather than teacher indicating approval or disapproval?	____	____
14.	listen to students?	____	____
15.	encourage students to verbalize their thinking?	____	____
16.	encourage students to defend or justify answers?	____	____
17.	encourage students to estimate and determine reasonableness of answers?	____	____

18. show more interest in the process of solving a problem rather than the answer? ____ ____

19. motivate students to want to think? ____ ____

20. give students time to think? ____ ____

21. offer activities which permit students to explore skills and/or concepts before trying to find a solution? ____ ____

INCORPORATING PROBLEM SOLVING SKILLS IN YOUR CLASSROOM

The more opportunities that students have for solving problems, the more skillful they will became as problem solvers. Time should be allotted during each class lesson to the skills of problem solving as well as to their application. Some suggestions are:

- Have students prepare their own original problems from a given set of data.
- Start each lesson with a problem solving experience.
- Be sure that each homework assignment contains at least one problem solving activity.
- Start a class book of problems. Bring in charts, menus, price lists, pictures, listings of information, graphs and tables gathered from newspapers and magazines. Ask pupils to write and solve problems related to these materials.
- Classroom activities such as collecting or spending money, team scores, cake sales, health data, planning for a trip or party opportunities for generating and solving verbal problems.
- Use student prepared problems for homework. Identify the problems with the student's name.
- Have students who wrote the problem check the answers submitted.

EFFECTIVE TEACHING OF PROBLEM SOLVING

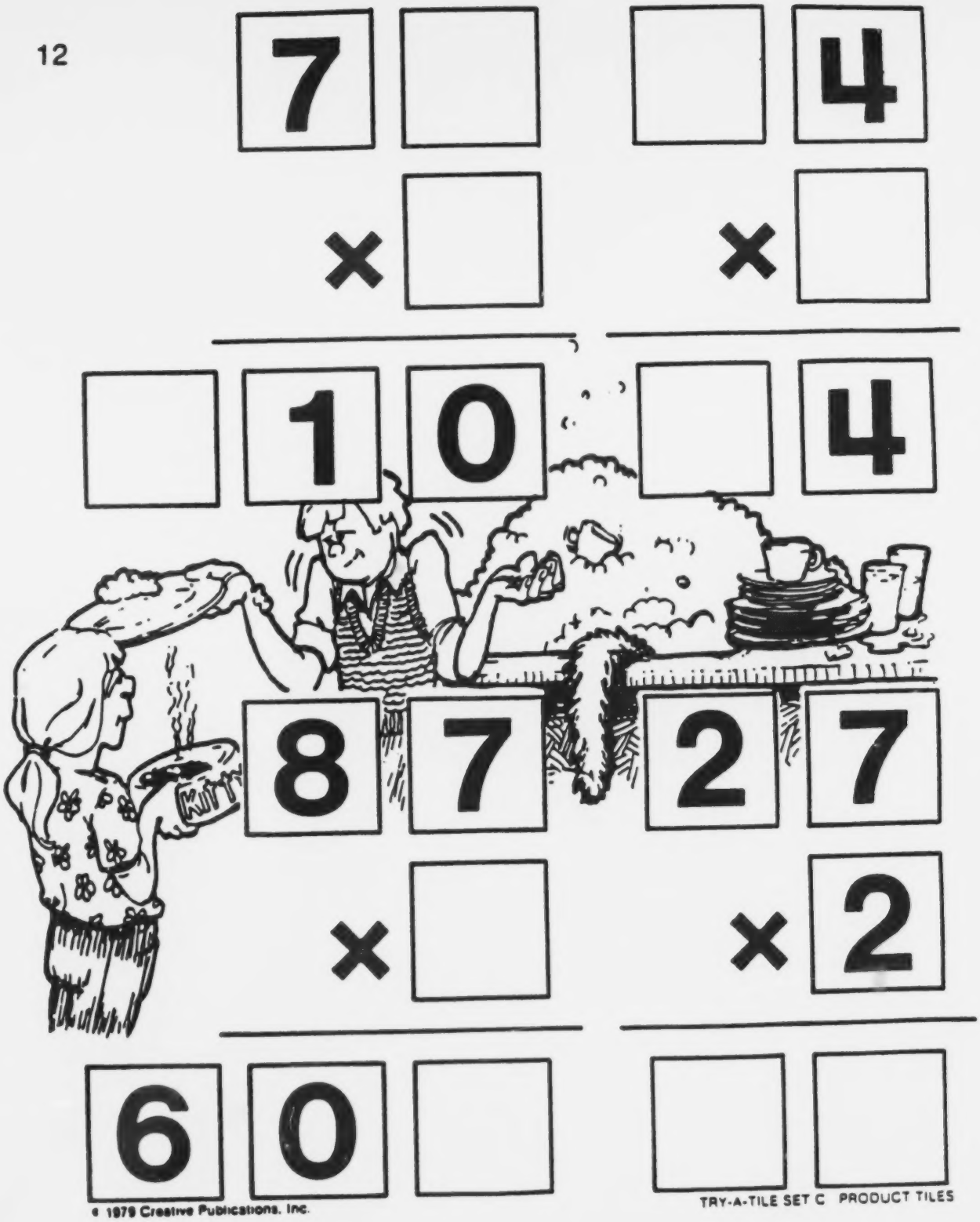
When visiting a classroom in which the learning of problem solving is taking place, one would expect the teacher to:

1. have carefully planned the lesson.
2. encourage students to verbalize their thinking.
3. ask a sequence of questions which lead students to discover a concept.
4. foster an exchange of ideas among students.
5. create the impression that the teacher is also investigating the problem.
6. encourage students to explore original ways of solving problems.
7. make clear that a variety of ways exist to solve problems by asking, "Can we think of another way of doing this?"
8. focus students' attention on the process of thinking.
9. promote divergent thinking.
10. avoid lecturing.
11. avoid asking "Are there any questions?" or "Is there anyone who doesn't understand?"
12. avoid allowing students to shout out answers spontaneously.
13. avoid using responses of more able students to dictate pace of class.
14. ask class to react to an answer given by a student rather than teacher indicating approval or disapproval.
15. encourage students to defend or justify answers.
16. encourage students to estimate and determine reasonableness of answers.
17. show more interest in the process of solving a problem rather than the answer.
18. motivate students to want to think.
19. encourage students to ask questions.
20. give students time to think.
21. listen to students.
22. offer activities which permit students to explore skills and/or concepts before trying to find a solution.

STRATEGIES FOR IMPROVING THE CLASSROOM CLIMATE FOR PROBLEM SOLVING

1. Emphasize ideas such as
   - problem solving takes time and patience
   - it is OK to make mistakes; sometimes errors help one arrive at a solution
   - a student who makes progress toward a solution has been successful
   - it's really fun and satisfying when you try hard and solve a problem
2. Give a story or a problem and have students act out the situation and phrase the question in their own words. Use objects when helpful.
3. Give students a number sentence and encourage them to formulate a problem that can be solved using the sentence. Post student's problems on a bulletin board for other students to solve, or make a weekly newsletter containing problems that students have created for others to solve.
4. Ask questions to help students understand the problem better.
5. Ask questions that give the students hints for solving when needed.
6. Ask questions that encourage the students to describe and analyze their solutions.
7. Compliment students for taking time to think through a problem rather than for how quickly they find a solution.
8. Use detective or mystery stories as an aid to help students problem-solving skills.
9. Look for everyday situations about which you can pose questions that stimulate thinking. e.g. At about what time will the hands on a clock be at right angles to each other?
10. Have students explain their plan and solution.
11. Have students look for and discuss alternate solutions to the same problem.
12. Help students build problem-solving skills by making new problems from a given problem. e.g. Change one or more of the conditions of the problem; interchange the "given" and the "to find".
13. Personalize problems. Use the names of your students and of local places to make problems come alive.
14. Encourage students to choose things they are interested in and write problems about them for other students to solve.
15. Have students work in groups and solve problems so that they can discuss the strategies possible to solve the problems.
16. Do not reject guesses;pursue them.

12



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TRY-A-TILE SET C PRODUCT TILES

Some Additional Problem Solving Strategies

Analyze Language: Problem solving often becomes an exercise in vocabulary. There are three classes of words or phrases which the teacher must be alert for:

a) words which come from mathematics only and have only technical meanings, such as sum, quotient, numerator, etc.,

b) words which have both technical meanings and broader meanings in everyday usage, such as product, square, area, interest, etc.,

c) key words and phrase which translate into mathematical concepts and notation, such as less than, greater than, equal to, increased by, decreased by, difference between, etc.

Efforts to lead students to understand the vocabulary of verbally stated problems will open up avenues of approach in solving them. Special vocabulary items of the kinds mentioned above should be listed with their meanings and reviewed and reinforced frequently. A special section of students' notebooks can be devoted to keeping an on-going glossary of terms encountered in verbal problems. Vocabulary items can be included in homework assignments and tests.

- Guess and Check: What number when multiplied by itself or when added to zero gives the same result? Have students guess numbers that seem to be possible answers and test them to see if they satisfy the conditions of the problem? Does 2 x 2 equal 2 + 0?

  Does the number 5 work? Does 1 work? Does 0 work? Students may discover that there is more than one such number. Or they may discover, in other problems, that there is no such number.

- Draw From Students' Background: "Mrs. Rodriguez wanted to buy a poound of potatoes costing $1.15, 2 pounds of steak at $3.50 a pound, and a cake for $1.50. Why did the store owner tell her that her $10 bill was not enough?" Lead students to recall actual visits to the supermarket. Ask if they ever had the experience of having to pay more for an item than the price that was marked on it.

- Make a Simpler Problem: "What is the largest number of rectangles that can be created by adding four line segments to a rectangle?" After agreeing that this is quite an involved question, have the class consider a more elmentary form of the same idea: "What is the largest number of rectangles that can be created by adding one line segment to a rectangle?" Then the class might consider two line segments, three and, finally four. Perhaps, before four line segments are finally considered, someone will have found a pattern and will be able to state a generalization for any number of additional line segments.

- Work Backwards: "Martin, Sandy and Alice each have some money. Martin gives Sandy two dollars and Alice one dollar. Then Sandy gives Martin three dollars and Alice one dollar. Finally, Alice gives Sandy two dollars and Martin one dollar. At the end of this process, Martin, Sandy and Alice each have twelve dollars. How much money did each of them start with?" This problem illustrates how organized thinking can lead smoothly to the solution of a problem which, at first, seems hopelessly confusing. In this case, the final outcome is known: each person ends up with twelve dollars. Beginning with this, it is possible to work backwards, step-by-step, to trace the amounts of money each person has at each of the four stages of the money exchange. The movement of dollars can be traced by working with a table such as the following, beginning at the bottom and working upward:

Martin	Sandy	Alice	
$12	$12	$12	(final situation)
$11	$10	$15	
$8	$14	$14	
$11	$12	$13	(amount of moeny each started with)

Notice that the total amount of money is $36. While the dollars change hands, the total number of dollars doesn't change.

PROBLEM SOLVING

In 1-12, tell whether you would use addition, subtraction, multiplication, or division to solve the problem in the shortest way.

1. If you know the cost of one shirt, find the cost of several such shirts.

2. If you know the depths of two lakes, find out how much deeper one lake is than the other.

3. A man travels the same number of miles to and from work on each working day. If you know this number of miles, find how far he travels on these trips during a 5-day work week.

4. If you know how much you paid for a given number of pounds of apples, find the cost of one pound of these apples.

5. If you know the length of a submarine and how much longer a cruiser is, find the length of the cruiser.

6. If you know the profits of a company that are to be shared equally among three partners, find the amount each

7. If you know the number of pairs of stockings in a box, find the number of pairs of stockings in a given number of such boxes.

8. If you know the number of pupils in a school and how many of these are boys, find the number of girls.

9. If you know how fast a car traveled and how much faster a plan flew, how fast did the plane fly?

10. If you know the rate of speed at which a ship is sailing and the number of hours it sails at this rate, find the distance it travels.

11. If you know how many copies of a newspaper are needed and how many copies are printed in one hour, find the number of hours needed to print the required number of copies.

12. If you know how much you agreed to pay for an article and how much you gave as a deposit, find the amount partner is to receive.you still have to pay.

PROBLEM SOLVING

Tell whether you would add, subtract, multiply, or divide to solve the problem.

1. Ruby, Britt, Lisa, Sean, Joan and Bill shared the expenses of a party. The party cost $24.72. How much was each one's share?

2. Jim bought a steak for $3.59, a bag of potatoes for $.79 and a loaf of bread for $.43. How much did he spend?

3. Joe bought a sweater for $12.99. He gave the cashier a $20 bill and 4 pennies. He received $7.05 change. Did Joe get the right change?

4. Mrs. Ling bought a 1-Kilogram can of coffee at a 25¢ off sale. The regular price was $6.14. How much did she pay?

5. Maria bought 5 kilograms of butter to make cookies. Butter was selling for $1.48 per kilogram. How much did Maria pay?

6. Apples were selling at 3 kilograms for $1.00. Mr. O'Neill bought 1 kilogram of apples. How much did he pay?

PROBLEM SOLVING

What else do you need to know before you can solve the problem?

1. A dozen golf balls are maked down $2.50. What is the cost per dozen?

2. A youth group washed 23 cars to earn money. How much did they earn?

3. Dog food is sold in 6-can packages. How much would 15 cans cost?

4. Wool material was on sale for $4.25 a meter. How much did it cost Dee to make a skirt?

5. A scout group needs $16.00 to go on a camping trip. How much will each scout have to give?

6. Doug's plane ticket cost $97. How much per kilometer did his trip cost?

7. Ms. Gomez drives 60 kilometers from her business to her vacation home. How long does it take?

8. Betty bought eggs for 87¢ and a bag of potatoes for $1.39. How much change did she get?

PROBLEM SOLVING

In 1-10, a fact needed to solve the problem has been omitted. State what this fact is.

1. Ruth paid 55 cents for a box of sugar. What was the cost of a pound of sugar?

   _____

   _____

2. If you bought two ties, how much did you pay for them?

   _____

   _____

3. Mr. Roberts saves the same amount of money each week in order to buy a television set. How long will it take him to save enough money to buy a television set that costs $190?

   _____

   _____

4. Sue bought a bag of potatoes for 60 cents. How much did she pay for a pound of potatoes?

   _____

   _____

5. If you paid $90 for 1 coat and 1 dress, what did you pay for the coat?

   _____

   _____

6. At 69 cents per gallon, what is the cost of a tankful of gasoline?

   _____

   _____

7. Harry took a trip in which his car averaged 15 miles per gallon of gas. How many gallons of gas did he use on the trip?

   _____

   _____

8. Mr. Wise bought an FM radio costing $120. He made a down payment of $30, and paid the rest in equal weekly payments. How many weekly payments did he have to make?

   _____

   _____

9. Mr. Breen spent 8 hours installing storm windows. On the average, how long did it take him to install one storm window?

   _____

   _____

10. Marian bought 6 pairs of stockings. How much change should she receive from a $20 bill?

    _____

    _____

PROBLEM SOLVING

What question could be asked about this information?

1. Mr. Overdraw wrote checks for these amounts:
   $192.50, $64.50, $18.53, $80.00, $13.64, and $83.50.

   Example answers: How much did Mr. Overdraw pay out?

   or

   How much more than $400 did he pay out?

   or

   What is the difference between the largest amount and the smallest amount?

2. The "Know It All Newspaper" has a daily circulation of 250,000 papers and a Sunday circulation of 325,000 papers.

3. Ms. Gray Eagle sold some clothing for $925. The clothing cost $560. Ms. Gray Eagle's overhead was $305.

4. The distance from the fulcrum to one end of a lever is 1 meter. The distance from the fulcrum to the other end of the lever is 15 centimeters.

5. Dot and Joe left Little Falls at the same time. Dot traveled north and averaged 72 kilometers per hour. Joe traveled south and averaged 64 kilometers per hour.

6. Mr. Kanelos paid for these items with a $10 bill.

Meat	$3.15
beans	.50
sugar	.42
cereal	.60
bread	.66

7. A mail carrier stops at 15 houses on a block. She walks 4 meters from the corner to the mailbox of the first house. She then walks 18 meters between the mailboxes of each house thereafter.

PROBLEM SOLVING

What information is not needed to solve the problem?

1. A bag of 5 grapefruit sold for 89¢. A bag of 6 oranges sold for 79¢. How much did Willie pay for 3 bags of oranges?

2. How much must Nadine pay for 3 pounds of grapes which come in 25-pound boxes and cost 49 cents per pound?

3. A car which weighs 3630 pounds travels at an average rate of 50 miles per hour. How far does it go in 4 hours?

4. How many shelves each 3 feet long can be cut from a 10-foot piece of lumber which costs $4?

5. Find the cost of a set of 12 napkins, each 16 inches by 16 inches, if the cost of each napkin is $3.

6. A salesman receives a weekly salary of $100. In addition he receives $3 extra for each article he sells. One week he sold 24 articles. How much extra did he receive that week?

7. Mrs. Basso's electric bill was $31.45. Her heating bill was $63.78, and her phone bill was $18.27. How much did Mrs. Basso spend on heat and electricity?

8. Carl left his home at 8:00 A.M. He averages 30 kilometers per hour and arrived at the ski lodge at 1:00 P.M. How long did the the trip take?

9. Rose left her house at 9:00 A.M. She averaged 25 kilometers per hour and drove for 8 hours. How far did she drive?

10. Amir is 1.6 meters tall and weighs 60.7 kilograms. Bill is 1.5 meters tall and weighs 80.6 kilograms. How much more does Bill weigh than Amir?

11. Debbie weighs 54.3 kilograms. She bought 5 kilograms of potatoes, 1.5 kilograms of meat, and .5 of a kilogram of butter. What was the total weight of the potatoes, meat, and butter?

12. Mr. Wolf's rent is $250 per month. His food bill averages $135 per month, and his commuting expenses amount to $30 per month. How much are his monthly rent and food expenses?

PROBLEM SOLVING

Enough, too much, or not enough information?

1. Who had the higher bowling score?

Game	Robert	Sidney
1	97	108
2	132	128
3	148	156

2. How much profit did the Bad Food Diner make?

Day	Receipts
Monday	$1,611.28
Tuesday	$1,247.55
Wednesday	$ 982.00

3. A piece of drill rod is 46 centimeters long. How many pins can be cut from it?

4. Ms. Regis used 200 liters of gasoline in July. She charged these amounts: $8.75, $10.50, $8.42, $3.58 and $7.24. What was the total cost of the gasoline?

5. Earth's diameter measures about twice as long as Pluto's diameter. What is the measure of Pluto's diameter?

6. Mrs. Brown died after paying 13 premiums of $65 each on an $8,000 life insurance policy. Mr. Brown received $8,000. The Browns had 3 children. How much more did he receive than the amount paid out?

7. Dora averages 19.2 kilometers on 3.8 liters of gasoline. How far can she travel on a full tank of gas?

8. Alex traveled 144 kilometers and used 19 liters of gasoline. How many kilometers per liter did he average?

HOMEWORK

Questions to consider:

a. How long?

b. Should it be cyclical?

c. Should it be graded in difficulty?

d. Should it be adapted for the level of the class?

e. When should it be prepared?

f. How is it given to the student?

g. What if there no text/poor texts?

h. Is it reviewed in class?

i. Are all problems reviewed?

j. How much time is spent in class reviewing the homework?

k. Who writes the problems on the board?

l. Who corrects the wrong solution?

m. Is it collected?

n. Is it returned?

o. What part of the grade does homework reflect?

p. Is parent notified of missing homework?

q. What about students who never do the homework?

ON HOMEWORK

HYPOTHESIS: Homework which reaches optimum significance needs to be carefully planned to harmonize with instructional goals. It should accent quality more than quantity. It should prevent wasted or uneconomical effort by students. It should be cyclical in nature to assure retention of previously learned material.

CHARACTERISTICS OF A POOR ASSIGNMENT:

1. The assignment consists of an arbitrary designation of pages or exercises in the text, chosen hurriedly and with little thought.

2. Prerequisite knowledge and skills have not been identified. There is no reason to believe the students can handle the material.

3. The assignment is fragmentary and bears little relation to what has gone before or what is to come after.

4. Although drill is not the goal, there is repetition to the point of frustration or boredom. No challenging exercises are present.

5. The assignment has not been planned for utilization in subsequent lessons; it doesn't set the stage for new ideas.

PLANNING VERSUS MAKING A HOMEWORK ASSIGNMENT:

Teachers should make a conscious effort in their planning:

a. to investigate future topics
b. to reinforce previous topics
c. to provide experiences that promotes discrimination among concepts in order to make the mathematics more available for the solution of the problems arising outside the mathematics classroom.

STEPS TO CONSIDER IN PREPARING CYCLICAL HOMEWORK ASSIGNMENTS:

1. The teacher should first list the topics that are to be taught each day (for the days between full-period tests).

2. The homework assignments are then planned <u>vertically</u> in the following manner: <u>ONLY TOPICS ARE LISTED FIRST.</u>
   a. the topic on the day's lesson is included for at least 3 successive days and then including it every other night or every third night, etc.
   b. The other topics chosen are from previous lessons alternating when they are given. Continuous computational topics should be included in most classes.

   c. After the topics have been listed for each assignment, the teacher should consider the length of the assignment and whether enough topics are covered for the needed variety on the test following these set of assignments. If adjustments are necessary, additional review from previous work can be included. Also movement of topics (because of too many long questions in one assignment versus not any long question, etc.) may be done at this point. Teachers are reminded that homework must be long enough to cover review but short enough to have students explain their solutions when it is gone over and yet allowing time for the teaching of a new developmental lesson each day.

d. After this planning, the books can now be used to select the problems. If this is done in a vertical fashion considering one topic and selecting problems for all of the assignments on this topic, gradation and variety on successive nights is made easier.

Each assignment should include at least 1 question that is verbal in nature.

Each assignment should contain in both single concept questions as well as multiple concept questions.

The new material should be lightly stressed in the homework. Giving 1 problem on the new topic is more successful than giving many. In this way, greater stress can be placed upon asking students to verbalize the rules, methods, or procedures needed to solve such a problem. If a student cannot do this problem, giving more than one problem will cause further frustration and math anxiety. If a student makes an error, he would reinforce this error many times if more than one problem is given. Giving one problem also provides a quick way of determining whether the new material was learned. Variations of the topic are given in successive nights.

e. Each assignment includes pages to be read on the topic taught that particular day.

f. Teachers should realize that if there is any uncertainty of completing a topic on a particular day, it is better not to include this topic on that night's assignment.

ADDITIONAL COMMENTS WITH REFERENCE TO HOMEWORK:

1. If homework assignments are planned from test to test,

   a. teachers will not only know whether they have enough topics for a varied and comprehensive test but will also know if they have both long and short topics for the given test.

   b. teachers can include review of topics previously tested that in their judgment need further review spirally in future homeworks and then include these topics on the next test. This should help the student who was unsuccessful on a given topic when it was first tested become confident when they are successful on the next test.

   c. students become more familiar with a comprehensive test and not as thrown when they take a state exam or a city-wide exam.

   d. the teacher can make up the test at the same time that they have prepared the set of homeworks between tests. It this way the test can be written, duplicated, proofread and a key prepared well ahead of it being given. Teachers are then directed as to what schedule of teaching should occur and in some way prevents going off on a tangent in class.

2. The overhead projector is very helpful in going over the homework. Students may purchase china markers at the beginning of the term. The teacher then gives an acetate when assigning problems to each student to bring in the following day. The student then comes to the machine, explains his/her work, chairing any questions asked from the class. This furthers verbalization by students and provides more sustained explanations. Teachers need to stress that students give explanations of what was one or thought of to go from step to step rather than have them read the work. No time is spent putting up the problems

on the board. All work becomes visable to the entire class. More student interaction takes place since this student is chairing questions from the class. Students may borrow the acetate and bring in the next day. If a disruption during a period (fire drill, etc.) the homework may be gone over the following day. Time may be spent on training students to pay attention to the discussion on hand not to copying of the work. More emphasis can be placed on re-doing work done incorrectly rather than mere copying of the solution. Going over the homework after the applications of the new lesson and before the summary of the lesson usually speeds up the going over of the homework rather than extending if to the entire period or most of the period.

3. Teachers are asked to prepare all solutions to each assignment in advance of their need with the students. In this way the teacher can determine the types of errors that could occur and know what to stress in the teaching of the topic. Time is saved since the teacher can quickly determine the point of error and ask questions accordingly.

4. Teachers may prepare homework sheets for distribution to the class. The benefits of having such a sheet are as follows:

   a. for students:
      (1) if absent, they have the homework and reading assignment and can come in prepared
      (2) provides information as to what topics to study for a test
      (3) gives enough advanced notice of test date to allow for planned study periods
      (4) if they know of another commitment for a particular night, they can do their homework in advance

   b. for teachers:
      (1) eliminates need for writing assignment on the board at the beginning of each class period
      (2) eliminates nightly pressure
      (3) eliminates having to first review which problems were already given
      (4) allows for gradation of problems
      (5) allows for a better planned test
      (6) provides material to be given to new admits and those students who return after extended absence.

   c. for parents:
      (1) gets better image of school, department and teacher since they see more planning
      (2) appreciates that child can do work if absent
      (3) appreciates not having last minute notice of test

   Sample model of information on homework sheet:

   #23 Given: Due:

Name of Text	104	201	204	212	277	313	Page #
Read 236-8,249, 250,305,306	46	15	48	13	19	16	Problem #

   Answers to problems:

   Note:

Putting answers on sheet helps student learn how to use answers when doing solutions and reduces math anxiety.

THE FOLLOWING MAY BE DISCUSSED WITH STUDENTS AND GIVEN TO THEM IN PRINTED FORM FOR REFERRAL AT HOME WHEN DOING THEIR MATHEMATICS HOMEWORK:

HOW TO DO YOUR MATHEMATICS HOMEWORK

PURPOSES OF HOMEWORK:

a. To help reinforce what you learned in class.
b. To help identify those parts of the lesson you do not understand.
c. To increase the number of correctly solved problems to which you can refer when reviewing for a test.

1. Read the introductory section of the assignment to identify and understand the principle it teaches.

   a. Make certain you understand the meaning of every word in the section.
   b. Visualize every step along the way.

2. Do the sample exercises.
   This will determine how well you understand the principles being taught and whether you will be able to work problems based on those principles.

   a. Read each step carefully in the model problems.
   b. Understand every term used.
   c. Follow the reasoning behind each manipulation.
   d. Solve the problem all over again on your own.

   It is a good idea to reserve a section of your loose-leaf notebook for working through the sample problems in the book.
   Writing out the steps is far better than merely reading through what the author has written.
   This section in your notebook will be a handy review for you before tests.

3. Recite the principles you have learned. If you cannot repeat the principles used, go back to the reading material prior to the exercises and read it through again.
   Write the principles and procedures used to solve each type of problem on an index card from which you can study for a test as well as quickly review previous work each day.
   Make review a part of your daily calendar. It will eliminate last minute cramming and further retention of your mathematical skills.

What to do if you "get stuck" on a problem!

a. Look back at the book, at your notes, and at the sample model problems.

What to do when an answer is wrong in your homework.

a. Analyze the problem to find out at what point the error was made.
b. Listen to the analysis done in class.
c. Take down the correct answer.
d. At home, re-do the problems you got wrong before trying the new assignment.

SAMPLE MF 1 HOMEWORKS

Date	Topic of Lessons	Current Topics				
	Reading A Ruler	Reading Ruler				
	Naming and Measuring		Measuring lines			
	Describing Changes by Directed Nos.	Reading Ruler		Changes as Dir. Nos.		
	Extension of Numbers Line to Integers		Measuring lines	Changes as Signed Nos. Number Line extended		
	Ordering Integers On Number Line	Reading Ruler		Changes as Signed Nos.	Ordering Integers	
	Ordering of Pos. and Neg. Rational Nos.		Measuring lines		Ordering Integers	Ordering Rational Nos.
	Comparing Rational Nos.	Reading Ruler		Changes as Signed Nos.		Comparing Rational Nos.
	Comparing Rational Nos. Contd			Study for Test		
	Test					

Review Topics

equivalent fractions	improper —> mixed	eval. formula		translation expressions	+ whole nos.
		eval. num. exp.	<or>		- whole nos. (verbal)
	mixed —> improper			translation expressions	x whole nos.
equivalent fractions		eval. formula	<or>		+ whole nos. (verbal)
	improper —> mixed	eval. num exp.			x whole nos. (verbal)
					+ whole nos. (verbal)
equivalent fractions		eval. formula			+ whole nos. (verbal)
	mixed —> improper		<or>		- whole nos.

POSSIBLE TOPICS FOR MF1 TEST

Reading a Ruler
Measuring Line Segments
Expressing changes as signed numbers
Identifying integers on the number line
Ordering integeers
Comparing Rational Numbers
Symbols of inequality
Evaluating numerical expressions
Evaluating algebraic expressions
Equivalent fractions
Expressing improper fractions as mixed numbers
Translation of statements into symbols
Whole Nos.--mult., division, add., subtr.(Computation and Verbal)

Sample Homeworks for Sequential Mathematics (MQ1)

Lesson Objective										Review Topics: x signed nos	+ signed nos	eval alg exp	+ mon	x mon	
Mult of a Polyl by a Monomial	Polyl x mon														
Product of Polyls using vertical form	Polyl x mon	Polyl x Polyl									± signed nos	eval alg exp	- mon	x mon	
Product of Polyls using Foil		Polyl x Polyl									+ signed nos (verbal)	eval alg exp		x mon	+ polyls
Division of Monomials	Polyl x mon	Polyl x Polyl	÷ mon								- signed nos	eval alg exp			- polyls
Negative and zero exponents			÷ mon	neg exp	zero exp						± signed nos	eval alg exp		x mon	+ polyls
Dividing Polyl x Mon	Polyl x mon	Polyl x Polyl	÷ like powers	neg exp	zero exp	polyl ÷ mon						eval alg exp			- polyl
Statement, Open Sentences			÷ mon	neg and zero exp		polyl ÷ mon	open sent				+ signed nos (verbal)			x mon	+ polyl
Translating Eng. Sentences into Algebraic Expressions		Polyl x Polyl	÷ like powers	neg and zero exp		polyl ÷ mon	open sent	translation				eval alg exp			± polyl
Solving x±a = b			÷ mon		zero exp		open sent	translation	solving x±a = b			eval alg exp		x mon	± polyl
Solving ax = b	Polyl x mon	Polyl x Polyl		neg exp		polyl ÷ mon		translation	solving x±a = b	solving ax = b	- signed nos (verbal)				± polyl

SAMPLE MQ 4 HOMEWORKS

Date	Topic of Lessons	Current Topics				
	Lesson 10-The Slope Formula	Slope formula				Dist. Form.
	Lesson 11-Slope of Parallel Lines	Slope formula	Slope parallel lines			Dist.
	Lesson 11-Slope of Perpendicular Lines		Slope parallel lines Slope perpendicular lines			
	Lesson 12-Graphing a Linear Equation		Slope perpendicular lines	Graphing Linear eqn. Point in a Line		Dist.
	Lesson 13-Point-Slope Form of the equation	Slope formula	Slope parallel lines	Graphing Lin. eqn.	Point-slope form of line	
	Lesson 14-Slope Intercept form of the Equation of a Line		Slope perpendicular lines	Point on a Line	Point slope form of line Slope int-form of line	Dist.
	Lesson 15-Proving Geometric Relationships		Slope parallel lines	Graphing Lin. eqn.	Point slope form of line Slope int-form of line	Dist. and proof
	Lesson 16-Proving Theorems Using Coord. Geom.		Study for Test			
	Test					

Review Topics

Midpt. form	Sine	Logic Quantifier	Geometric Proof • Triangles Less than.	
	Tangent			When is Quad. a parallelogram
Midpt.		Logic Quantifier	Geometric Proof • Triangles Converse isoceles thm.	
	Cosine			When is Quad. a rhombus
Midpt		Logic Quantifiers		
	Tangent			When is Quad. a rect.
Midpt.				When is Quad. a square

POSSIBLE TOPICS FOR MQ4 TEST:

Slope Formula
Slope and Parallel Lines
Slope and Perpendicular Lines
Finding the slope of a line
Writing the equation of a line using point-slope form and slope-intercept form
Distance Formula
Midpoint Formula
Logic: Negation, Quantifiers
Use of trigonometric functions
Graphing a linear equation
Determining if a point is on a line
Determining type of quadrilateral
Geometric proofs--congruence
Proving geometric relationships using coordinate geometry

Homework

Analyze the homework assignment given in the lesson plan for the lesson on "The angles of a triangle". (p. 105)

- What features of this homework assignment indicate a lack of planning on the part of the teacher?
- How interesting do you think this assignment would be to students?
- To what degree do you think this homework assignment incorporates cyclical review questions?
- How can this assignment be improved? Specifically:

  What writing activities might be included?

  How can student-designed problems be included?

  How would you review this assignment in the next lesson?

  What non-routine activities might be included?

Situations for Discussion by New Teachers: Homework

1. You discover, by looking over the first batch of homework papers, that one student writes fives and N's backwards and has generally very poor handwriting. What do you do?

2. While speaking to you about the several homework assignments that she has not done, a student explains that she can't do homework, since she works every afternoon and has heavy family responsibilities in the evening. What do you do?

TESTS

Questions to consider:

a. When should tests be given?

b. How much notice should students be given?

c. What is to be tested?

d. What types of questions should be given?

e. How long should a test be given?

f. How difficult should the test be?

g. What strategies should be given to the student to study for the test?

h. What should be looked at while making a test?

i. How do you use the test results?

j. Should a test be reviewed in class after it is marked? When? How?

k. How does future homework relate to test results?

TESTING

1. **PLANNING OF TESTS:**

   a. Tests should be planned ahead. A worthwhile suggestion is to plan the next test as soon as the previous one is given after planning the homework assignments from one test to another.
   b. The teacher should begin by listing the topics to be tested as a result of the homework planning and classwork experiences.
   c. The teacher should consider the objectives of each question.

2. **SOURCE OF QUESTIONS:**

   a. Same type as homework.
   b. Same type as done in class.
   c. Use of other teacher-made tests from the past.
   d. Use of test booklets prepared by publishers.
   e. Use of previous state exams or school uniform exams.
   f. Use of texts.

3. **LENGTH OF TEST:**

   a. The teacher should take the test first, then multiply the time by about $5\frac{1}{2}$.

      Correct test length if necessary keeping in mind that time is needed for distribution and collection. The time factor by which one multiplies may vary depending upon the type of class that is being tested, the amount of reading involved with each question, etc.

4. **TYPE OF QUESTIONS:**

   a. short questions that are single-concept questions, multi-concept questions, verbally stated questions, etc.
   b. multiple choice questions
   c. long questions

5. **FORMAT OF TESTS:**

   a. The test should start off with easy questions, go to difficult and then to the more difficult questions i.e., ascending order of difficulty.
   b. Topics covered on last two days before the test should not be included on the test since students need to practice these topics through homework.
   c. Standards from State Education Department should be used in both the wording and the selection of test questions.
   d. Students should be informed as the point value on the test and the directions to be followed so that they may budget their time accordingly.

5. **FORMAT OF TESTS CONTINUED:**

e. Space should be allowed on tests for students to show work in an organized fashion.
f. Homework should be planned cyclically so that a frequent review of topics covered on the test may be done through the homework.
g. Tests should include "extra for experts" question(s) in which no actual points are given for correct work but an indication such as a "+" next to the grade. This "+" may be considered when determining a student's grade for the marking period or course.
g. Tests should be neat and not crowded. It is suggested that a draft of the actual test be written first, then changes made if necessary.
h. Duplication of the test should be done prior to the day of need so that the test may be taken by the teacher, proofread ahead of time so that students are not spending unnecessary time on a problem in which an error occurs, and a marking key may be made up as the proofreading is done.

6. **MARKING OF TESTS:**

a. Standards of marking as used in marking state exams should be used for marking of teacher-made tests.
b. A marking key should be used so that uniformity of grading takes place.
c. In marking a question where partial credit is to be given, the teacher should decide before marking how many credits are to be given to each step in the solution. If a problem contains several parts of which some are dependent on others, the teacher should not penalize the student several times but should give him credit if his work for one part is consistent with his work on the other part. i.e., if a student "follows through correctly", deduction is made only for the initial error made.
d. It is also suggested that if there are any long questions, the teacher should mark the same question on each paper first and then mark the second long question, etc. In this way the marking will be consistent on all student papers.
e. If an answer is wrong, put an x through the answer so that it cannot be changed without being noticed.
f. Besides putting the grade on the test, whenever possible try to write a comment that would encourage better performance or give credit to the students' work.
g. If comments concerning organization of students' work is warranted, include them either with the individual question or with the grade at the top of the test.
h. Try to have the tests marked by the day after it is given so that the greatest gains can occur in learning from mistakes which are "fresh in students minds". Do not go over a test unless the students have their papers in front on them.
i. As you mark the papers, keep a list of those students who you would want to write their solutions on the board when the test is gone over. (It is suggested that you have an alternate student in case of absence). One student may be chosen to do two or possibly three questions at the board depending upon the length of the questions.

j. An analysis of test results should be done so that you are aware of the type of errors that have been made so that emphasis can be made on the particulars in your future teaching.
k. It is NOT necessary to inform the students of the number of students failing, etc. (This type of discussion should be avoided with the students since it usually causes more problems than being helpful to the improvement of instruction).

7. **USE OF TEST RESULTS:**

a. An item analysis should be made. There should be a definite follow-up in which notes are made on the difficulty of each question.
b. The length of the test should be analyzed.
c. Comments should be made for future references.
d. An analysis of test results should be made for evaluation of success of various work taught. This analysis should make the teacher aware of what points to stress when the test is gone over.
e. Successive homework assignments should then include questions that caused more difficulty to students. This should be done through a spiral review of topics. These items may then be retested on the next test.
e. Students organization of work should be analyzed so that specific strategies can be given to students to improve their organization.
f. Common errors made by students should be determined to make changes in future teaching of these topics.

8. **RETURNING OF TESTS:**

a. Tests should be returned the very next day after it is given if possible. In this way it is fresh in the student's mind as to what he/she did on the test. It also shows the students that the teacher is concerned about the improvement of learning in a sequential subject.
b. Test papers should be arranged in order by rows to facilitate the distribution of the papers. The teacher, as he/she is marking the papers, should make a list of those students who should put up the work considering their work shown and not just the correct answer. If several approaches to the same problem appear, each method should be put up at the board. If no students get a particular question correct, the teacher should elicit the necessary steps in the solution by asking questions directed toward that goal.
c. A "Do Now" on the day of going over the test might be any of the following:
    a. Having students explain, in writing, definitions, rules or procedures for topics from the test.
    b. Having students explain, in writing, a comparison between the procedures needed for different topics from the test.
    c. Review of the pre-requisite skills needed for the next lesson.

    The "Do Now" should not be similar problems to those causing difficulty on the test since if they couldn't do the problems and did not have another explanation of the solutions, they would not be able to do the "Do Now" exercises.

d. A discussion should take place with the students (prior to any distribution of test papers) on the purposes of going over a test. e.g. looking for one's point of error looking for alternative methods

Students must feel that they have something to gain from this experience and that understanding what they got wrong has value for the future especially in our sequential subject matter.

Sttudents must be informed as to the need for taking notes on problems whether they were right or wrong.

e. The board should be marked with the problem numbers and the students' names of those responsible for putting up the complete solution. Emphasis must be placed on the need for work shown that would be helpful to those students having difficulty with each problem. This assignment of students should have been done at the time of marking the papers.

f. Before distributing the papers, students should be told that if their name appears on the board that they are writing the work for the benefits of their classmates.

g. As students are writing on the board, the teacher should be walking around discussing individual papers with students, making suggestions to them, encouraging them individually, and checking that students are looking at the boardwork and taking notes.

h. Students are then to go to the board and explain (not just read) their work to their classmates. If questions arise, they should should be directed to the entire class for responses.

i. Strategies can be discussed with students on budgeting of time on a time, preventing careless errors, organizing work, techniques of studying for a math test, etc.

j. Students should be allowed to keep their test papers and stress should be placed on studying from them for future exams.

SUGGESTIONS FOR STUDENTS RELATED TO MATHEMATICS EXAMS

To the teacher: What additional suggestions would you give to students?

1. Start reviewing far enough in advance so you have time to do a careful, unhurried job.

2. Review by RE-DOING model problems, homework problems, class problems, problems from previous tests, and quizzes.

3. When you take a test, go in with an attitude that will help you. Have confidence in your own ability, but don't be over-confident. Don't try to "get by" with doing as little as possible. Don't worry to the point of anxiety. Fear alone can make a person do poorly on a test regardless of his ability and knowledge.

4. Follow directions. Read carefully. Consider the point value.

5. Look over the whole test quickly at the start and do the ones you are sure of first.

6. If you are unable to answer a question, leave it and go on to others, coming back to the hard ones later.

7. Be careful to show clearly what you are doing. Put in enough steps so that your method is perfectly clear.

8. Work neatly. It fosters accuracy.

9. Be organized in your solutions. It helps reduce the errors that could occur.

10. Check back for accuracy as you go along.

11. If you have extra time at the end of the test, re-check your work.

TAKING A MATH TEST

STRATEGIES THAT CAN HELP THOSE WHO CAN DO PROBLEMS IN CLASS BUT NOT ON THE TEST

1. Each day copy an assortment of problems on index cards. On the front of the card write the question, and on the reverse side write the complete solution. At least twice a week, shuffle the deck of index cards and deal yourself the number of problems you can complete in one hour. Work the problems and grade yourself by comparing your solutions with the ones found on the reverse side of the index cards. For each problem you fail to complete successfully, find at least two similar problems in your index cards, notes, or book. Complete and check these additional problems during the same reviewing session.
2. By placing your questions on index cards, you are providing a situation that closely resembles test conditions. That is, verbal hints from section titles are not easily accessible and the questions are in random order.
3. Re-doing rather than reading problems will show you whether you really know the mathematics.

STRATEGIES THAT CAN HELP THOSE WHO MAKES MANY CARELESS ERRORS ON A TEST.

1. Organization is important. It helps reduce the errors that could occur.
2. Work neatly. It fosters accuracy.
3. Check back for accuracy as you go along.
4. If you have extra time at the end of the test, re-check your work by re-doing each question.
5. Check your answers for reasonableness.
6. Re-read each question after you have completed your work to be certain you have answered the particular question.
7. Do not write over numbers. Rewrite numbers when changes occur.

STRATEGIES THAT CAN BE OF HELP TO THOSE STUDENTS WHO GET NERVOUS DURING A MATH TEST.

1. Start reviewing for enough in advance so you have time to do a careful, unhurried job.
2. Assume that the teacher will give a quiz tomorrow on what you've studied today.
3. When you take a test, go in with an attitude that will help you. Have confidence in your own ability, but don't be over-confident. Don't try to "get by" with doing as little as possible.
4. Don't worry to the point of anxiety. Fear alone can make a person do poorly on a test regardless of his ability and knowledge.
5. Do the questions that you are sure of first.
6. Get a good night's rest the night before an examination.

STRATEGIES THAT CAN HELP THE STUDENT WHO RUNS OUT OF TIME ON A TEST

1. Look over the whole test quickly at the start and do the ones you are sure of first.

2. Give careful thought to the first steps of the solution, write them out in detail at your normal pace. Mistakes made in the first few steps are usually penalized more heavily, since they may indicate a complete lack of knowledge on how to begin the problem.
3. Construct drawings that can improve your presentation of the problem and its solution. Record all formulas or theorems that seem to apply to the problem.
4. After completing two or three steps in the solution, quickly determine if you can complete the solution--in the limited time remaining--at a faster pace.
5. If you do not believe that enough time is available, outline the steps you would take to complete the problem without actually writing out the required statements, or computations. You are at least showing the teacher that you have enough knowledge to complete the problem.

CHECKLIST FOR EVALUATING A MATHEMATICS TEST

		Yes	NO
1.	Are there directions for taking the test?	___	___
2.	Are the directions simple and clear?	___	___
3.	Is the test legible?	___	___
4.	Is the test of appropriate difficulty?	___	___
5.	Is the test of appropriate length?	___	___
6.	Is space provided for showing work?	___	___
7.	Are answer spaces provided?	___	___
8.	Are point values indicated?	___	___
9.	Is duplication avoided? Is each concept tested only once?	___	___
10.	Do short questions test only one mathematical concept?	___	___
11.	Is there provision for writing extended responses?	___	___
12.	Is there provision for original problem solving as well as algorithmic exercises?	___	___
13.	Is there provision for extra credit for students who finish early?	___	___
14.	Is there a variety of types of problems on the test?	___	___
15.	Is there provision for reinforcing previously learned skills?	___	___
16.	Does the test include review items based on previous homework and class exercises?	___	___

MF1 Test #2 NAME _____

DIRECTIONS: Show all work to receive full credit. Questions 1-33 are 3 pts. each; Bonus: 1 point

1. ADD: 6 + 3,439 + 29

2. MULTIPLY: 789 x 678

3. SUBTRACT 678 FROM 2,011

4. DIVIDE: $16\overline{)14688}$

5. Change $3\frac{7}{16}$ to an improper fraction

6. Change $\frac{29}{5}$ to a mixed number

7-12 Write each phrase or statement in symbols.

7. 12 subtracted from w

8. y divided by 29

9. The sum of 32 and 24 subtracted from 205

10. Fifteen added to the product of x and y

11. Twelve is greater than nine.

12. Thirty is less than four times sixty.

13-15 Supply the missing number to make each statement true.

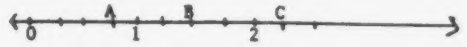
13. $\frac{17}{30} = \frac{\quad}{180}$ 14. $\frac{4}{7} = \frac{16}{\quad}$ 15. $\frac{9}{\quad} = \frac{18}{16}$

16-20 Write the signed number described by each of the following expressions.

16. 5 units to the right of +5 ____
17. 9 units to the left of +2 ____
18. A withdrawal of $100 ____
19. 17 ft. below sea level ____
20. change in temperature from -2 to +5 ____

21-23 Refer to the number line and write the numbers represented by:

21. A _____ 22. B _____ 23. C _____



24-27. Find the value of the following numerical expressions:

24. 8x3 + 2x6 25. $2^3 + 3^2$

26. 25-(3·2+5) 27. $(6+4)^3$

28-29. Evaluate each algebraic expression using a=7, b=9, c=8

28. $14 - \frac{b}{9}$ 29. a(b+c)

30. Use your ruler to find the length of the following line segment

_____

31-33. Place the symbol "<", ">" or "=" to make each statement true

31. 3+4·8+(10-2) ____43

32. $\frac{3}{4}$ ___ $\frac{1}{2}$ 33. $\frac{3}{16}$ ___ $\frac{1}{8}$

EXTRA FOR EXPERTS:
Find the number halfway between $\frac{2}{5}$ and $\frac{3}{5}$

SEQUENTIAL MATH 1 TEST NAME _____

All work must be shown to receive credit.

QUESTIONS 1,4,7,& 8 are 1 pts each; all other questions are 4 pts. each

1. If the sum of two numbers is negative,
   (a) their product is also negative
   (b) both numbers are negative
   (c) at least one number is negative

2. Evaluate: $\frac{(+9)+(-5)}{(-2)}$

3. One morning in November, the temperature at the base of Mt. Washington was 35° above zero. At the top of the mountain, it was 10° below zero. What was the difference in temperature?
   (a) 25° (b) 45° (c) 55° (d) not given

4. Which of the following is an open sentence?
   (a) x + 2 (b) x + 3=4 (c) 5 + 4=9 (d) 12>10

5. "Four more than twice a number" can be expressed by which of the following?
   (a) 2(x+4) (b) 2x > 4 (c) 4(2x) (d) 2x+4

6. Using "n" to represent the number, represent the following in terms of "n".
   "When 7 is subtracted from a number, the result is 9:"

7. Which of the following is a pair of like monomials?
   (a) $8x$ & $9x^2$ (b) $8x^2y$ & $9x^2y$
   (c) $8x$ & $9y$ (d) $8y$ & $9y^2$

8. $2x^3$ is equivalent to
   (a) $2x \cdot 2x \cdot 2x$
   (b) $2 \cdot x \cdot x \cdot x$
   (c) $2 \cdot 2 \cdot 2 \cdot x \cdot x \cdot x$
   (d) $2 \cdot 2 \cdot 2 \cdot x$

9. If x=8, the value of x-5(x+2) is
   (a) -42 (b) -22 (c) 30 (d) 42

10. If x= -2, find the value of x^2-4x

11. Express as a single power of x:
    $x^4 \cdot x^2 \cdot x$

12. Express as a single power of x:
    $\frac{(x^2)^2}{x^3}$

13. An expression equivalent to $(3k^3)^2$ is
    (a) $9k^6$ (b) $27k^6$ (c) $27k^5$ (d) $9k^5$

14. Simplify: $3r^2 + 2r^2$

15. Simplify: 13w-8w+5w

16. (8y)-(-3y) =

17. $(3x^3y)(-4xy^4)$ =

18. $\frac{40b^3x^6}{-8b^2x}$

19. Find the sum of $2x^3-3x-4$ and x^3-2x+3

20.- 23. Perform the indicated operations and express each answer in simplest form:

20. $(9y^2+4y-1)-(5y+2)$ =

21. $2x^2(3x+4xy)$ =

22. $(3x+5)(2x-7)$ =

23. $\frac{9x^3-6x^2+3x}{3x}$ =

24. If $y=4x^0+x^{-2}$, find the value of y when x=5

25. Solve algebraically for y:
    47 = y + 13

26. If p - 8 = 3, find the value of 5p

EXTRA FOR EXPERTS:

(a) Express in simplest form:
$\frac{6(-7+5)-4(-1-1)}{-2(9-13)}$

(b) If $k^x=y$, then k^{x+2} equals
(1) y+2 (2)2y (3) y^2 (4)k^2y

MG3 TEST 2 NAME: _____ (last) _____ (first) Row: ___ Seat: ___ A

All work must be shown on this paper to receive full credit. No separate scrap paper is allowed. Use blue or black ink. Ques 1-6,9-16 5pts ea; 7,8 6pts ea.

1. A postulate is a statement which is
   1) sometimes proved
   2) always proved
   3) accepted without proof
   4) accepted without definition _____

2. If a triangle has sides measuring 8", 6", and 8", then the triangle is classified as a(an) _____ triangle.

3. What type of angle is an angle that contains 200° ?
   1) acute
   2) reflex
   3) obtuse
   4) straight
   ____

4. Find the measure of each base angle of an isosceles triangle if the vertex angle measures 30°.
   _____

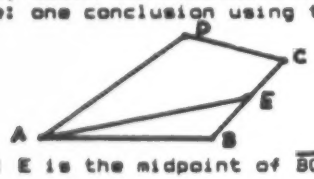
5. A, B, and C are collinear points and point C is between A and B. If AC=9 and CB=3, find AB.
   ____

6. In the diagram below, which angle is adjacent to $\angle CED$?

   1) $\angle BEA$
   2) $\angle DEA$
   3) $\angle EDA$
   4) $\angle EDC$ ____

In 7-8, state two conclusions that can be drawn from the given data for each figure: one conclusion using the symbol $\cong$, the other using the symbol $=$.

7.

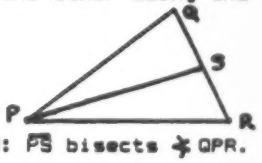


GIVEN: E is the midpoint of $\overline{BC}$.

CONCLUSIONS: _____ $\cong$ _____

_____ $=$ _____

8.

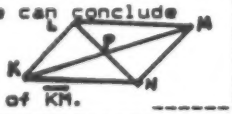


GIVEN: $\overline{PS}$ bisects $\angle QPR$.

CONCLUSIONS: _____ $\cong$ _____

_____ $=$ _____

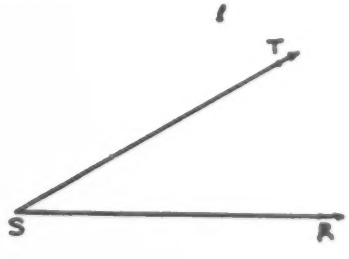
9. If $\overline{LP} \cong \overline{NP}$, then we can conclude
   1) $\overline{KP}$ bisects $\overline{LN}$.
   2) $\overline{LN}$ bisects $\overline{KM}$.
   3) $\overline{KP}$ bisects $\angle LKN$.
   4) P is the midpoint of $\overline{KM}$. _____



10. If $WZ = ZY$, then what type of line segment is $\overline{XZ}$?
   1) altitude
   2) angle bisector
   3) median _____

In 11-12, Use only a compass and a straight edge. Be sure to leave all construction lines visible.

11. Construct an angle at B that is congruent to $\angle RST$.



12. Construct a line segment that will bisect $\overline{CD}$.

13. Which is logically equivalent to $\sim(p \wedge \sim q)$?
1) $p \wedge \sim q$
2) $\sim p \wedge q$
3) $\sim p \vee q$
4) $\sim p \vee \sim q$ _____

14. Write the following sentence in symbols:
There exists an integer x, such that x is less than seven.

15. Write the negation of $\exists_x : x+4 \neq 8$

16. Which is the negation of the statement "No horses are green."
1) Some horses are green.
2) All horses are green.
3) Some horeses are not green.
4) All horses are not green.

_____

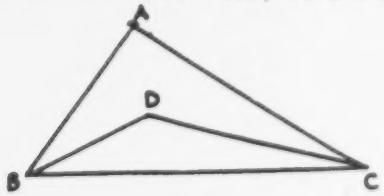
17. GIVEN: If Peter dates Amy, then Ben will be jealous.
If Peter dates Carol, then Ben will not be jealous.
Peter dates Carol or there will be a fight.
Peter dates Amy.

Let A represent "Peter dates Amy."
Let B represent "Ben will be jealous."
Let C represent "Peter dates Carol."
Let F represent "There will be a fight."

a) Using A,B,C,F, and proper connectives, express each sentence in symbolic form. [4 pts]

b) Prove: There will be a fight. [14 pts]

EXTRA FOR EXPERTS: In $\triangle ABC$, the bisectors of $\angle ABC$ and $\angle ACB$ meet at point D. If $m\angle BDC = 130$, find $m\angle BAC$.



MQ4 TEST . Name: _____ Row: ___ Seat: ___
last first

All work must be shown on this paper to receive full credit. Ques 1-12, 15 5pts
Separate scrap paper is not allowed. Use blue or black ink. 13, 14 4pts

1. The negation of $k \vee \sim m$ is
   1) $\sim k \vee \sim m$
   2) $\sim k \wedge \sim m$
   3) $k \wedge \sim m$
   4) $\sim k \wedge m$

____

2. Write a valid conclusion for the premises below:
Either the shoe fits or I won't wear it.
The shoe does not fit.

_____

3. Find the value of x.

_____

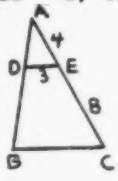
4. Which of the following CANNOT be the lengths of the 3 sides of a right triangle?
1) 1, $\sqrt{3}$, 2
2) 5, 7, 8
3) 3, 4, 5

_____

5. Corresponding sides of 2 similar triangles are 6 and 9. If the perimeter of the larger triangle is 36, find the perimeter of the smaller triangle.

_____

6. If $\overline{DE} \parallel \overline{BC}$, DE = 3, AE = 4, and EC = 8, then find BC.



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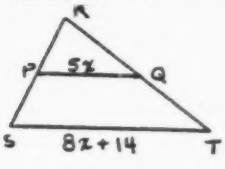
7. P and Q are the midpoints of RS and RT, respectively.
If PQ = 5x and ST = 8x + 14, find the value of x.

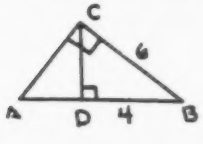


_____

8. The altitude to the hypotenuse of a right triangle divides the hypotenuse into two segments. If the altitude measures 6" and one segment of the hypotenuse measures 12", find the length of the other segment.

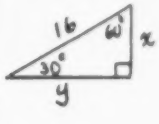
_____

9. $\triangle ABC$ is a right triangle and $\overline{CD}$ is the altitude drawn to hypotenuse $\overline{AB}$.
If CB = 6, and DB = 4, then find AD.



_____

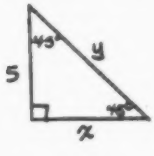
10. Find the value of x and y.



x = _____

y = _____

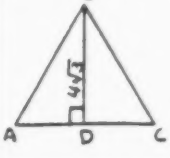
11. Find the value of x and y.



x = _____

y = _____

12. In equilateral triangle ABC, altitude BD = $4\sqrt{3}$. Find AB.



_____

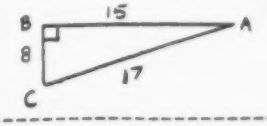
13. a) Find the value of cos 58°

_____

b) Find m∠G if sin G = .5878

_____

14. As a fraction, write the value of



a) tan C _____

b) sin A _____

Continued

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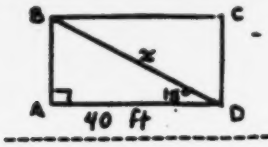
15. Write an equation that can be used to find the value of x. [DO NOT SOLVE]



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16. Find to the nearest degree the measure of the angle of elevation of the sun when a 3 ft. vertical pole casts an 8 ft. shadow. [10 pts]

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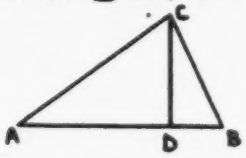
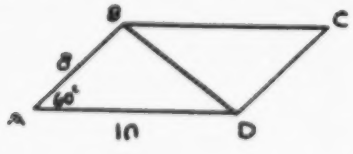
17. In △ABC, $\overline{CD}$ is the altitude to $\overline{AB}$, AB = 40, AC = 30 and m<A = 35. [17 pts]



a) Find CD to the nearest TENTH.

b) Using your answer in (a), find the area of △ABC.

Extra for experts: GIVEN: ▱ABCD with AB = 8, AD = 10, and m<A = 60. Find BD (Answer may be left in radical form).



MQ4 TEST 2 Name: _____ _____ Stat. ___
last first

All work must be shown on this paper to receive full credit. Separate scrap paper is not allowed. Use blue or black ink. | Ques. 6-9 3pt ea 1-5,10-14 5pt ea

1. The negation of "All cats are not green." is
1) All cats are green.
2) No cats are green.
3) Some cats are green.
4) Some cats are not green.
_____

2. In the domain of real numbers, which of the following is true?
1) $\exists_x$: $x - 3 = 7$
2) $\forall_x$: $x - 3 = 7$
3) $\forall_x$: $x - 3 \neq 7$
_____

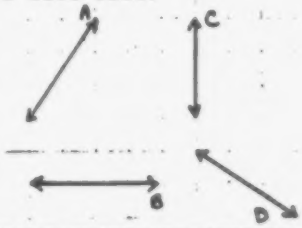
3. Find the length of the line segment joining (4,3) and (1,-2).
_____

4. Find the coordinates of the midpoint of the line segment whose endpoints are (3,6) and (7,5).
_____

5. Point B is the midpoint of AC. Their coordinates are A (2,1), B (5,3), and C (8,k). What must be the value of k?
_____

For 6-9, in the spaces, write the letter of the line with the slope described.

___ 6. positive slope
___ 7. negative slope
___ 8. zero slope
___ 9. no slope



10. Determine if the points A (3,3), B(1,2), and C(-5,-2) are collinear. (Justify your answer).
_____

11. The slope of $\overleftrightarrow{AB}$ is $\frac{2}{3}$ and the slope of $\overleftrightarrow{CD}$ is represented by $\frac{6}{x+1}$.
Find the value of x. so that $\overleftrightarrow{AB}$ will be parallel to $\overleftrightarrow{CD}$
_____

12. The slope of $\overrightarrow{RS}$ is $\frac{3}{4}$. If $\overleftrightarrow{RS} \perp \overleftrightarrow{TV}$, find the slope of $\overrightarrow{TV}$.
_____

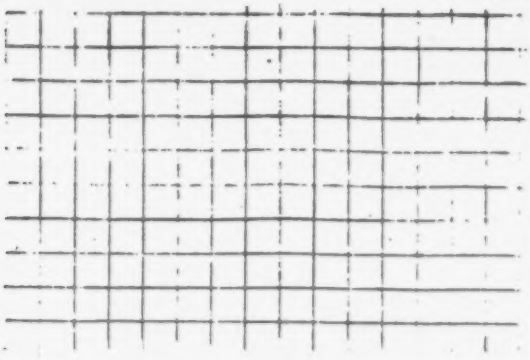
13. Which of the following points lie on the graph of $3x - 2y = 6$?
1) (4,-3) 2) (3,4)
3) (-2,-6) 4) (0,6)
_____

14. The equation of the graph of a line with a y-intercept of -2 and parallel to the x - axis is?
1) $x = -2$ 2) $y = x - 2$
3) $y = -2$ 4) $y = -2x$
_____

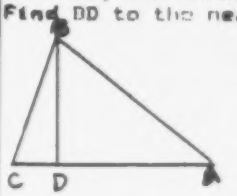
15. Write the equation of a line with a slope of 3 and passing through (1,4)
_____

Continued

16. Graph $2x + y = 1$ [8pts]

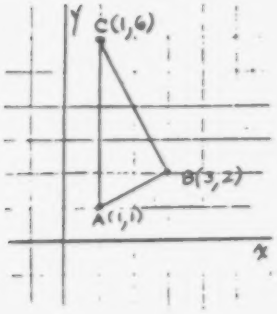


17. In $\triangle ABC$, $\overline{BD}$ is the altitude to $\overline{AC}$, AB = 20, and $m\angle ABD = 35$. Find BD to the nearest TENTH. [6pts]

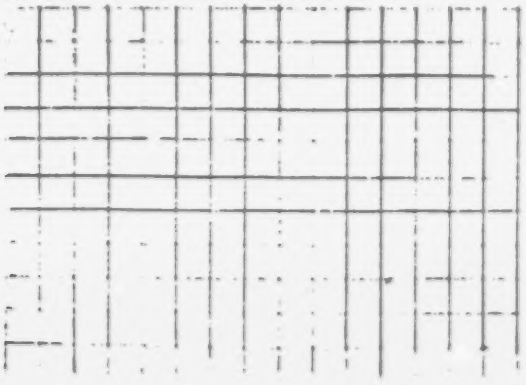


18. a) Find the length of each side of $\triangle ABC$. [10 pts]

b) Show $\triangle ABC$ is a right triangle. (Be sure to also state a reason) [7 pts]



EXTRA FOR EXPERTS: Given pts R(4,3), S(7,5), and T (8,1). In $\triangle RST$, $\overline{RV}$ is the altitude drawn to $\overline{ST}$. Write the equation of line $\overleftrightarrow{RV}$.



Testing

Use the Checklist for Evaluating an Exam as a tool for evaluating the sample exam given in this manual.

- What items on the exam should be eliminated?
- How would you change the items on the exam?
- What additional items would you include?
- What are the advantages of providing point values?
- How would you improve the physical appearance of the exam?
- How could you provide for original problem solving on the exam?

Situations for Discussion by New Teachers: Testing

1. You give an exam on a unit of work and discover that 18 of your 30 students failed it. What do you do?

2. A student comes to you after class and complains that she understands the work, but simply can't do well on tests. What do you do?

Student Assessment

For the four workshop participants nearest to your seat, write their names below and assign each a grade for class participation, based on their performance during this staff development session. The grades that you may use are:

E +
E Excellent
E -
G +
G Good
G -
S +
S Satisfactory
S -
U Unsatisfactory

Participant's Name	Class Participation Grade

- What criteria did you use in assigning your grades?
- If any of the grades that you gave were contested, how would you be able to substantiate them?
- How did the grades that you gave compare with the grades that other participants gave for the same individuals?

GENERAL CRITERIA FOR EVALUATION OF S. 'DENTS IN MATHEMATICS CLASSES

1. The teacher would do well to base his grade on as ..uch objective evidence he can assemble because he/she will have to justify a grade to himself, to his students, and sometimes to parents and supervisors. He/she can do this more readily through the objective evidence he/she has of a student's achievement than through his subjective evaluation of many intangible traits.

2. An absolute standard should be used and the grade a student receives should be determined by the degree to which the student has met the absolute standard.

3. Every consideration should be given to allow students to make up work missed for excusable absence. Attendance, lateness and cutting are to be considered in determining a student's grade.

4. Make should NOT be used as a form of punishment.

5. Marks should NOT be influenced by the "halo effect."

6. After a tentative grade is determined using the grading system stated below the teacher should now review the progress made up by each individual (i.e., did his grades go up during the year or term as well as considering extenuating circumstances (i.e., illness on day of test).

SAMPLE GUIDELINES FOR COMPUTING GRADES

Keep in mind that each report card mark represents a cumulative evaluation of the student's achievements.

Since grades are an individual matter, it is difficult to establish iron-clad rules to govern every situation. However, to insure uniformity and fairness for all of our students, the following guidelines have been established to provide direction for arriving at grades.

CLASS TESTS: 90%

Each test should include:

60% on work that can be done by most students similar to classwork and homework problems.

20% on work that is somewhat more taxing

20% on work that only superior students can do.

CLASS AND HOMEWORK: 10%

Includes teacher's judgment on participation in class activities, work at desks, attentiveness to class activities, demonstration at the board, responses to questions (which need not always be perfectly correct), preparation of homework, completeness of homework, and quality of student's notebook.

GENERAL SUGGESTIONS ON EVALUATION OF STUDENTS

1. Explain your grading policies to the students -- eliminates misunderstandings and discontent among the students.

2. Be certain that the students ..derstand the directions on any test you give.

3. State your questions on any exams clearly using terminology appropriate to the student's level of understanding.

4. Base your tests on important material that has been taught thoroughly and reviewed cyclically through homework.

5. Use a prepared answer key in marking your tests to standardize your grading.

6. DO NOT allow personal feelings to enter into a grade.

7. You are NOT responsible for passing every student, but you are responsible for giving every student a fair chance and for trying to help students who are not doing well.

Situations for Discussion by New Teachers: Student Assessment

1. A candidate for graduation comes to you after final report cards in June and pleads for you to change his failing grade to a passing one, since yours is the only grade preventing him from meeting his diploma requirements. What do you do?

2. An underachieving student has just shown you some very substandard work of which she is very proud.

3. A student is often absent from class, sometimes without excuse, never does homework, but gets 100's and 90's on every exam and quiz. It is time to formulate grades. What do you do?